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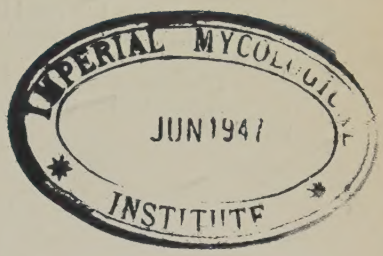

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MATHEMATICS

ON AN EFFECTIVE METHOD OF SOLVING EXTREMAL PROBLEMS FOR QUADRATIC FUNCTIONALS

By L. V. KANTOROVITCH

(Communicated by V. I. Smirnov, Member of the Academy, 14. VI. 1945)

We shall consider here a method of successive approximations applicable to a wide class of minimum problems for quadratic functionals. This method may be used for actually solving these problems as well as for their qualitative study (existence theorems, properties of the solutions, *etc.*).

Let us outline the general idea of the method. A functional $I(f)$ on a linear metric space is considered. $I(f)$ is supposed to be quadratic, that is $I(f+\varepsilon g)$ is a polynomial of the second degree in ε. Given a fixed element f_0 we choose the direction of the gradient of $I(f)$ at $f=f_0$, *i. e.*, the element $g=g_1$ such that $[d/d\varepsilon \cdot I(f+\varepsilon g)]_{\varepsilon}=0 \,\|\, g \,\|$ has an extremal value (the space may be, for instance, of type (F) [1]). Further we obtain $\varepsilon=\varepsilon_1$ from the condition that $I(f_0+\varepsilon g_1)$ should attain an extremum, and then the process is to be repeated again and again.

Let us consider the application of the method to some concrete cases.

1. Systems of Linear Algebraic Equations. Let a system

$$\sum_{k=1}^{n} a_{i,k}\, x_k = b_i \quad (i=1, 2, \ldots, n) \tag{1}$$

be given. Consider the form

$$H(X)=\sum_{i=1}^{n}\Big(\sum_{k=1}^{n} a_{i,k}x_k - b_i\Big)^2 \tag{2}$$

which attains the minimum equal to zero at $X=\{x_i\}$ equal to the solution of (1). Suppose $x_i=x_i^0$ are initial values of x's. We take

$$H(X^{(0)}+\varepsilon Z)=\sum_i\Big[\sum_k a_{i,k}(x_k^0+\varepsilon z_k)-b_i\Big]^2=H(X^{(0)})+$$
$$+2\varepsilon\sum_k\Big(\sum_i r_i^{(1)}a_{i,k}\Big)z_k+\varepsilon^2\sum_i\Big(\sum_k a_{i,k}z_k\Big)^2 \tag{3}$$

where

$$r_i^{(1)}=\sum_k a_{i,k}x_k^{(0)}-b_i$$

The coefficient of 2ε is maximal when $\sum z_k^2=\text{const}$ if z_k are proportional

to their own coefficients. We put simply

$$z_k^{(1)} = \sum_i a_{i,k} r_i^{(1)} \tag{4}$$

To minimalize $H(X^{(0)} + \varepsilon Z^{(1)})$ we set

$$\varepsilon = \varepsilon^{(1)} = -\frac{\sum_k z_k^{(1)2}}{\sum_i \left(\sum_k a_{i,k} z_k^{(1)}\right)^2} \tag{5}$$

Now, we put

$$r_i^{(2)} = \sum_k a_{i,k} (x_k^{(0)} + \varepsilon^{(1)} z_k^{(1)}) - b_i = r_i^{(1)} + \varepsilon^{(1)} \sum_k a_{i,k} z_k^{(1)} \tag{6}$$

and then calculate successively $z_k^{(2)}$, $\sum_k a_{i,k} z_k^{(2)}$, $\varepsilon^{(2)}$, $r_i^{(3)}$ *etc.* by the formulae analogous to (4), (5), (6). The solution of (1) is

$$x_i = x_i^{(0)} + \varepsilon^{(1)} z_i^{(1)} + \varepsilon^{(2)} z_i^{(2)} + \ldots \tag{7}$$

Computations may be arranged in a simple scheme and can easily be carried out by calculators. The formulae and the computations can be essentially simplified in case the matrix $||a_{i,k}||$ is symmetric.

The method of successive approximations just considered converges (*i*) to the solution if the latter is unique, (*ii*) to one of the solutions if there are several, (*iii*), to the solution of the least square problem if there is no solution.

In particular, it is convenient to apply the method to normal systems when using the Gauss least square method. The order of convergence equals that of a geometrical progression (usually with a small denominator).

Let us notice the simple geometrical interpretation of the method. Namely, given a family of similar ellipsoids $H(X) = \text{const}$, we draw the normal to the ellipsoid passing through the initial point $X^{(0)}$. Then we choose another ellipsoid which touches the normal at point $X^{(1)} = X^{(0)} + \varepsilon^{(1)} Z^{(1)}$, draw the normal at this point and so on.

We notice finally that the method may be applied to infinite systems as well.

2. Fredholm's Integral Equations. For the sake of simplicity we consider only the case of symmetric kernel.

The solution of the equation

$$L(\varphi) = \varphi(x) - \lambda \int_a^b K(x, y)\, \varphi(y)\, dy - h(x) = 0 \tag{8}$$

extremalizes the integral

$$H(\varphi) = \int_a^b \varphi^2(x)\, dx - \lambda \int_a^b \int_a^b K(x, y)\, \varphi(x)\, \varphi(y)\, dx\, dy - 2 \int_a^b \varphi(x)\, h(x)\, dx \tag{9}$$

provided $\lambda/\lambda_k < 1$ $(k = 1, 2, \ldots)$.

Proceeding from an arbitrary $\varphi_0(x)$ we construct the successive approximations in the same way as above; we put

$$\varphi_1(x) = \varphi_0(x) - \lambda \int_a^b K(x, y)\, \varphi_0(y)\, dy - h(x) \tag{10}$$

$$\varepsilon_1 = -\Big(\int_a^b \varphi_1^2(x)\,dx\Big) : \Big(\int_a^b \varphi_1^2(x) - \lambda \int_a^b \int_a^b K(x,y)\,\varphi_1(x)\,\varphi_1(y)\,dx\,dy\Big) \qquad (11)$$

The following functions $\varphi_k(x)$ and numbers ε_k are defined analogously. The solution $\bar{\varphi}(x)$ is given by the formula:

$$\bar{\varphi}(x) = \varphi_0(x) + \varepsilon_1\varphi_1(x) + \ldots \qquad (12)$$

If $\varphi(x)$ is a solution of (8), which necessarily has a solution, λ being no-eigenvalue, then, as is easily seen,

$$H(\varphi_0) - H(\varphi_0 + \varepsilon_1\varphi_1) =$$
$$= \Big(\int_a^b \varphi_1^2(x)\,dx\Big)^2 : \Big(\int_a^b \varphi_1^2(x)\,dx - \lambda \int_a^b \int_a^b K(x,\ y)\,\varphi_1(x)\,\varphi_1(y)\,dx\,dy\Big)$$

$$H(\varphi_0) - H(\bar{\varphi}) =$$
$$= \int_a^b \eta^2(x)\,dx - \lambda \int_a^b \int_a^b K(x,y)\,\eta(x)\,\eta(y)\,dx\,dy; \quad \eta(x) = \varphi_0(x) - \bar{\varphi}(x)$$

Hence if α_k are Fourier's coefficients of $\varphi_1(x)$ with respect to the eigenfunctions,

$$[H(\varphi_0) - H(\varphi_0 + \varepsilon\varphi_1)] : [H(\varphi_0) - H(\bar{\varphi})] =$$
$$= \Big(\sum \alpha_k^2\Big)^2 : \Big[\Big(\sum \Big(1 - \frac{\lambda}{\lambda_k}\Big)\alpha_k\Big)\Big(\sum \Big(1 - \frac{\lambda}{\lambda_k}\Big)^{-1}\alpha_k^2\Big)\Big]$$

It is evident that if α and β are any numbers satisfying the inequalities

$$0 < \alpha \leqslant 1 - \frac{\lambda}{\lambda_k} \leqslant \beta < +\infty$$

we have

$$H(\varphi_0) - H(\varphi_0 + \varepsilon_1\varphi_1) \geqslant \frac{\alpha}{\beta}[H(\varphi_0) - H(\bar{\varphi})]$$

We see that the functional tends to its minimum with the rapidity of a geometrical progression. Hence follows that the series (12) converges as rapidly (more accurately: the denominator of the progression does not exceed $(\beta-\alpha)^2 : (\beta+\alpha)^2$).

The method is also applicable to the case where λ is an eigenvalue, when it is required to find the eigenfunctions and the eigenvalues. It may also be applied to equations with non-symmetric kernels, as well as to the case where the condition $\lambda/\lambda_k < 1$ is not fulfilled. In the later case we have to consider $S[L(\varphi)]^2 dx$ instead of $H(\varphi)$. Finally, the method may be used in solving equations of the 1st kind.

3. Ordinary Differential Equations.

Consider the equation

$$L(y) = \frac{d}{dx}(p(x)\,y) - q(x)\,y - f(x) = 0; \quad y(a) = y(b) = 0 \qquad (13)$$

To (13) there coorresponds the minimum problem for the integral

$$I(y) = \int_a^b [p(x)\,y'^2 + q(x)\,y^2 + 2f(x)\,y]\,dx \qquad (14)$$

Taking a function $z(x)$ which satisfies together with $y_0(x)$ the condition $z(a)=z(b)=0$ we easily obtain

$$I(y_0+\varepsilon z)=I(y_0)+2\varepsilon\int_a^b\left[py_0'-\int(qy_0+f)\,dx\right]z'\,dx+$$
$$+\varepsilon^2\int_a^b(pz'^2+qz^2)\,dx$$

To find $z=z_1$ extremalizing the factor at 2ε under the condition $\int z'^2\,dx=\text{const}$ we put

$$z_1'=p'y'-\int(qy_0+f)\,dx+C \text{ or } z_1''=L(y_0) \tag{15}$$

With z_1 obtained from this equation we put, correspondingly to the conditions $z(a)=z(b)=0$

$$\varepsilon_1=-\left(\int z'^2\,dx\right):\left(\int[p(x)z'^2+q(x)z^2]\,dx\right) \tag{16}$$

and, finally,

$$y_1(x)=y_0(x)+\varepsilon_1 z_1(x) \tag{17}$$

The further approximations can be determined in a similar way. For the case where $p(x)>0$, $q(x)\geqslant 0$ the following inequality holds, analogous to that in section 2,

$$I(y_1)-I(y_0)\geqslant\frac{p_{\min}}{p_{\max}+\frac{(b-a)^2}{\pi^2}q_{\max}}[I(y_0)-I(\bar{y})] \tag{18}$$

where $\bar{y}$ is a solution. Hence follows that the approximations converge to the solution and the convergence has the rapidity of a progression.

It is possible to apply the method to equations of higher orders, as well as to other boundary problems. It can also serve as a basis for various grapho-analytical methods. Thus beam designers will find it useful in reducing the design of trussed beams to a repeated calculation of simple beams.

4. Boundary Problems for Partial Differential Equations. Consider, for instance, the Dirichlet problem for the self-adjoint elliptical equation

$$L(u)=\frac{\partial}{\partial x}\left(a\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(b\frac{\partial u}{\partial y}\right)-cu-f=0;\ u=\varphi(s) \text{ on } \Gamma \tag{19}$$

The minimum problem for the integral

$$I(u)=\iint_D\left[a\left(\frac{\partial u}{\partial x}\right)^2+b\left(\frac{\partial u}{\partial y}\right)^2+cu^2+2fu\right]dx\,dy \tag{20}$$

is associated with (19).

Taking a function u_0 which assumes the assigned values on the boundary Γ of the domain D as the initial approximation, and a function η equal to zero on Γ, we can obtain

$$I(u_0+\varepsilon\eta)=I(u_0)-2\varepsilon\iint_D L(u_0)\,\eta\,dx+$$
$$+\varepsilon^2\iint_D\left[a\left(\frac{\partial u}{\partial x}\right)^2+b\left(\frac{\partial \eta}{\partial y}\right)^2+c\eta^2\right]dx\,dy.$$

It is natural to put $\|\eta\|=\iint_D[(\partial\eta/\partial x)^2+(\partial\eta/\partial y)^2]\,dx\,dy$. Then, under the

condition $\|\eta\| = \text{const}$, the factor at 2ε attains its extremum when $\eta = \eta_1$ is replaced by the solution of the equation

$$\Delta\eta_1 = L(u_0); \quad \eta_1 = 0 \text{ on } \Gamma \tag{21}$$

As to $I(u_0 + \varepsilon\eta)$ it is minimalized by $\varepsilon = \varepsilon_1$ where

$$\varepsilon_1 = \left[\iint_D L(u_0)\,\eta_1\,dx\,dy\right] : \left\{\iint_D \left[a\left(\frac{\partial\eta}{\partial x}\right)^2 + b\left(\frac{\partial\eta}{\partial y}\right)^2 + c\eta^2\right] dx\,dy\right\} \tag{22}$$

Then $u_1 = u_0 + \varepsilon_1\eta_1$ will be the first approximation of u. The convergence of the process can also be proved under certain conditions.

Here the practical use of the method is hampered by the necessity to solve Poisson's equation (when determining η) at every step of the process. Thus the Green function of the domain D is required. But seeing that the domain can be transformed into a circle (or a sphere in the case of spatial transformation) by suitably changing the variables without altering thereby the type of the equation and that the Green function of the circle is elementary, it appears that in a number of cases our method may also find practical application.

On its basis different numerical, graphical and experimental methods can be developed. It can also be applied to other types of equation and to other graphical problems.

5. Functional equations in Hilbert's space. Let H be a self-adjoint positive-definite operator defined on a linear manifold R_1 in the Hilbert space R. Let T be an analogous operator which takes R_1 onto R and possesses the inverse T^{-1} on R. We suppose H to be bounded with respect to T, that is, $0 < \alpha(Tf, f) \leqslant (Hf, f) \leqslant \beta(Tf, f)$. Let us find a solution $f \in R_1$ of the equation

$$Hf - \varphi = 0 \tag{23}$$

A solution $\bar{f}$ of this equation minimalizes the quadratic functional

$$I(f) = (Hf, f) - 2(f, \varphi) \tag{24}$$

Let $g \in R_1$ and let $f_0 \in R$ be the initial value of f. Then

$$I(f_0 + \varepsilon g) = I(f_0) + 2\varepsilon(Hf_0 - \varphi, g) + \varepsilon^2(Hg, g)$$

The second summand in the right-hand side attains its maximum, under the condition $(Tg, g) = \text{const}$, when $Hf_0 - \varphi$ is proportional to Tg, in other words, when g is proportional to

$$g_1 = T^{-1}(Hf_0 - \varphi) \tag{25}$$

Having chosen the element g_1 we minimallize I by making ε equal to

$$\varepsilon_1 = -(Hf_0 - \varphi, g_1) : (Hg_1, g_1) = -(Tg_1, g_1) : (Hg_1, g_1)$$

Put now $f_1 = f_0 + \varepsilon_1$. Then we obtain

$$[I(f_1) - I(f_0)] : [I(f_0) - I(\bar{f})] = \frac{(Tg_1, g_1)^2}{(Hg_1, g_1)\frac{1}{\alpha}(H(f_0 - \bar{f}, f_0 - \bar{f})} =$$

$$= \frac{(Tg_1, g_1)^2 (Hf_0 - H\bar{f}, f_0 - \bar{f})}{(Hg_1, g_1)(Tg_1, f_0 - \bar{f})^2} \geqslant \frac{(Tg_1, g_1)^2 (Hf_0 - H\bar{f}, f_0 - \bar{f})}{(Hg_1, g_1)(Tg_1, g_1)(Tf_0 - T\bar{f}, f_0 - \bar{f})} \geqslant$$

$$\geqslant [(Tg_1, g_1)(Hf_0 - H\bar{f}, f_0 - \bar{f})] : [(Hg_1, g_1)\frac{1}{\alpha}(Hf_0 - H\bar{f}, f_0 - \bar{f})] \geqslant \frac{\alpha}{\beta}$$

It is evident that $I(f_n)$ tends to $I(f)$ with the rapidity of a geometrical progression. Hence we may conclude that $f_n \to f$ in the sense of the norm $\|g\| = (Tg, g)$.

It is possible to apply the method to non-quadratic functional (for instance, to systems of non-linear algebraic equations). In such cases, however, the determination of ε becomes essentially more difficult, and we have to content ourselves with approximate values of ε.

It may be remarked in conclusion that this method though evolved here independently, is connected with the author's general concepts regarding extremal problems. These ideas are briefly outlined in [2].

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[1] S. Banach, Théorie des opérations linéaires, Warszawa, 1933. [2] L. Kantorovitch, C. R. Acad. Sci. URSS, XXVIII, No. 3 (1940).

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MATHEMATICS

ON TABULATING A CLASS OF INEXPLICIT FUNCTIONS OF FOUR VARIABLES

By L. NEUSCHULER

(Communicated by S. L. Soboleff, Member of the Academy, 20. XI. 1944)

It often occurs that we have to tabulate a function $v = f(x, y, z, u)$ defined by the equation

$$\varphi[\eta(v, x), y] = \psi[\mu(v, z), u] \qquad (A)$$

where $\frac{\partial}{\partial v}\left[\frac{\mu(v, z)}{\eta(v, x)}\right] = 0$, and φ and ψ are arbitrary tabulated functions of two variables, η and y, respectively μ and u.

Let

1) x assume the sequence of values $a_1, a_2, \ldots, a_m$,
2) y » » » » » $b_1, b_2, \ldots, b_n$,
3) z » » » » » $c_1, c_2, \ldots, c_p$,
4) u » » » » » $d_1, d_2, \ldots, d_q$,
5) $\alpha_2 \leqslant \alpha = \eta(v, x) \leqslant \alpha_2$;
6) $\beta_1 \leqslant \beta = \mu(v, z) \leqslant \beta_2$;
7) $\mu(v, z) / \eta(v, x) = \gamma(x, z)$.

Let us now insert between the smaller of the two numbers α_1 and β_1 (suppose that $\alpha_1 < \beta_1$) and the greater of the two numbers α_2 and β_2 (suppose that $\beta_2 > \alpha_2$) a sequence of terms of the geometrical progression with the ratio $1+\varepsilon$, where ε corresponds to the required accuracy of the values of v.

We now construct

1) m tables $A_1, A_2, \ldots A_m$, in the i-th of which on the k-th «place» (the places are supposed to be ordered) we insert the value of v satisfying with the required accuracy the equation $\eta(v, a_i) = \alpha_1(1+\varepsilon)^{k-1}$;

2) n tables $B_1, B_2, \ldots B_n$, in the j-th of which on the same k-th place we insert the value $\varphi[\alpha_1(1+\varepsilon)^{k-1}, b_j]$;

3) q tables $C_1, C_2, \ldots C_q$, in the l-th of which on the r-th place (the places in the tables $C_1, C_2, \ldots, C_q$ are supposed to be ordered in the same way as in the Tables A_i and B_j) we insert the value $\psi[\alpha_1(1+\varepsilon)^{r-1}, d_l]\, u$;

4) the table D with two entrances x and z of the values of the function $N = \mathrm{E}\lg_{(1+\varepsilon)}\gamma(x, z)$*.

We shall show, how it is possible by using the tables A_i, B_j, C_l and D to determine the values of v for given x, y, z, u, *i. e.* to solve the equation (A).

* The sign E denotes the absolute value.

Suppose we have to determine v from the equation (A) for $x=a_i$, $y=b_j$, $z=c_s$ and $u=d_l$.

This may be done in four steps:

1) We start with the table D, by which we determine $N=E\lg_{(1+\varepsilon)}\gamma(a_i, a_s)$. Consider two cases: $N>0$ and $N<0$.

If $N>0$, we proceed as follows:

2) Mark in the table C_l the place N.

3) Find among the numbers (the values of φ) situated in the table B_j on the t-th place ($t=1, 2, \ldots$) and, correspondingly, in the table C_l (the values of ψ) on the $(t+N)$-th place equal ones, and mark this place in the table B_j. Let this be the g-th place.

4) On the same g-th place in the table A_i we find a number which is the required value of v.

In fact, we have found in the table A_i on the g-th place the value of v satisfying with the required accuracy the equation $\eta_i(v, a_i)=\alpha_1(1+\varepsilon)^{g-1}$. The number M found on the same g-th place in the table B_j is the value of $\varphi[\eta(v, a_i), b_j]$.

On the other hand, the number equal to N found on the $(g+N)$-th place in the table C_l is the value of $\psi[\eta(v, a_i)(1+\varepsilon)^N, d_l]$, but $\mu(v, c_s)/\eta(v, a_i)=\gamma(a_i, c_s)$, and $E\lg_{(1+\varepsilon)}\gamma(a_i, c_s)=N$. Consequently, the value of v found in the table A_i on the g-th place satisfies the equation

$$\varphi[\eta(v, a_i), b_j]=\psi[\mu(v, c_s), d_l].$$

If $N<0$, then we proceed in the same way, as in the first case with the only difference that we look for equal numbers in the tables B_j and C_l among the numbers situated in the table C_l on the t-th place ($t=1, 2, \ldots$) and, correspondingly, in the table B_j on $(t+|N|)$-th place.

We have considered the case of tabulating the equation (A) under the condition $\partial/\partial v[\mu(v, z)/\mu(v, x)]=0$, *i. e.* when the ratio $\eta(x, z)/\eta(v, x)$ does not depend on v.

Let us consider now the second case, when the difference $\mu(v, z)-\eta(v, x)=\gamma_1(x, z)$ does not depend on v.

In this case we proceed in the following way:

1) We insert between α_1 and β_2 not a geometrical, but an arithmetical progression with the difference d (determined by the required accuracy of the values of v).

2) We construct the corresponding tables A_i and B_j (in which the «places» correspond to the subsequent terms of the arithmetical progression) and

3) We construct the table D not for the values of $E\lg_{(1+\varepsilon)}\gamma(x, z)$, but for the values of $\gamma_1(x, z)/d$.

The schedule of operations with these tables remains the same as in the first case. To the case $N>0$ here corresponds the case $\mu(v, z)-\eta(v, x)>0$ and to the case $N<0$, the case $\mu(v, z)-\eta(v, x)=\gamma_1(x, z)<0$.

Remark 1. To make the performance of the operation 3) more convenient it is advisable to have the tables C_l bound in a separate booklet, since it will then be more convenient to look for equal among the corresponding numbers of the tables B_j and C_l.

Remark 2. It is easily seen that for the solution of the equation (A) we may replace m (m being the number of tabulated values of the argument x) tables $A, A_2, \ldots, A_m$ by p (p being the number of tabulated values of the argument z) tables A_i ($i=1, 2, \ldots, p$) writing in them in the k-th place the value of v satisfying with the required accuracy the equation $\mu(v, c_i)=\alpha_1(1+\varepsilon)^{k-1}$ instead of $\eta(v, a_i)=\alpha_1(1+\varepsilon)^{k-1}$. Correspondingly, in the operation 4) in the solution of the equation (A) the requried v will be determined not by the table A_i, but by the table A_i'. In the mode of operation this second variant does not differ from the considered first variant. But it may differ in the volume. Namely, its volume will be smaller or greater

than the volume of the first variant according as p is smaller or greater than m. On this criterion variant of the smaller volume is chosen.

Remark 3. The tables A_i, B_j and C_l may be given an arbitrary form with the only restriction that the places in them should be ordered in the same way. Practically, two forms are to be considered: the form of rectangles and the form of columns.

The form of columns is to be preferred in the case where the number of the terms of the progression inserted between α_1 and β_2 or, which is the same thing, the number of different values of given accuracy assumed by the intermediary functions η and μ is such that it occupies a table page of handy size.

Let m, n, p and q, *i. e.* the numbers of tabulated values of the arguments x, y, z and u, be respectively equal to 10, 20, 40 and 30, and the number of different values of given accuracy assumed by the functions η and μ be 100.

Thus the table for the solution of the equation (A) may be profitably arranged as follows (for a small degree of accuracy of the values of N, v, φ and ψ the table will occupy only one sheet of handy size):

z \ x	I $a_1 \dots a_i \dots a_{10}$	II $a_1 \dots a_i \dots a_{10}$	III $b_1 \dots b_s \dots b_{20}$	IV $d_1 \dots d_l \dots d_{30}$	
c_1 c_2 · · · c_{40}	N	v	φ	ψ	(B)

Suppose we have to determine v for $x=a_i$, $y=b_s$, $z=c_l$.

1) In part I of the table we determine N from a_i and b_s.

2) Compare the numbers of the t-th line ($t=1, 2, \dots$) of the column b_s with the respective numbers of the $(t+N)$-th line of the column d_l (this for $N>0$; for $N<0$ we proceed *vice versa*), till we find in some L-th line of the column b_s and the $(L+N)$-th line of the column d_l equal numbers.

3) In the intersection of the L-th line with the column a_i we find in the part II of the table the required v.

If the number of the tabulated values of the arguments denoting the columns of the parts II, III and IV* of the table (B) is such that it occupies more than the breadth of the sheet, then we subdivide these values into groups and construct tables of the type (B) (without the part I) for all possible combinations of these groups, subjecting this subdivision to the condition that the general volume of the tables should be least. The part I is then to be given separately.

We have considered the cases of tabular solution of the equation (A), when the ratio or the difference of the functions $\eta(v, x)$ and $\mu(v, x)$ does not depend on v. It may be easily shown that in a more general case, where it is possible to find for the functions $\eta(v, x)$ and $\mu(v, z)$ such a function F that

$$F[\eta(v, x), \mu(v, z)] = \gamma(x, z)$$

* As regards the number of the values of φ and ψ we assume that in height they occupy not more that a sheet of the given size.

does not depend on v, a convenient tabular solution of the equation (A) is also possible.

Suppose that

1) from $F[\eta(v, x), \mu(v, z)] = \gamma(x, z) = N$ follows $\mu(v, z) = F^*[\eta(v, x(N]$;

2) the functions $\eta(v, x)$ and $\mu(v, z)$ assume the sequence of values $M_1, M_2, \ldots, M_s$, and

3) the function $N = \gamma(x, z)$ assumes the sequence of values $N_1, N_2, \ldots, N_r$, and x, y, z, and u assume the same values as in the first case.

Construct:

1) The table I for the function $N = \gamma(x, z)$.

2) The table II consisting of the tablets $A_1, A_2, \ldots, A_m$ in the i-th of which on the r-th place we write the value of v satisfying with the required accuracy the equation $\eta(v, a_i) = M_r$.

3) The table III consisting of tablets $B_1, B_2, \ldots B_n$ in the j-th of which on the same r-th place we write the value of $\varphi(M_r, b_j)$.

4) The table IV consisting for every value of $N_\alpha (\alpha = 1, 2, \ldots, k)$ of q tablets $C_1, C_2, \ldots, C_q$, in the l-th of which on the r-th place we write the value of $\psi[F(M_r, N_\alpha), d_l]$. The determination of the value of v from the values of x, y, z and u in equation (A) may be performed by means of the tables I — IV in the following way:

1) In the table I we find $M = \gamma(x, z)$ from the values of x and z.

2) Step by step, starting with the «first» value of φ in the tablet (table III) marked by the given value of y we compare these values with the values of ψ (in the same «places») situated in the tablet (table IV) marked «N» and «u», till on some Q-th place we find equal numbers (with the required accuracy).

3) In the same Q-th place in the tablet (Table II) marked by the given value of x we find the required v.

The constructions considered here may as well be carried out graphically.

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MECHANICS

DETERMINATION OF THE EFFICIENCY OF A TOOTHED-GEAR MECHANISM WITH MANY DEGREES OF FREEDOM

By M. KREINES

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Let us consider, just as we have done in a previous paper (1), a certain toothed-gear mechanism K, consisting of several elementary mechanisms K_1, K_2, ..., K_n each of whose planet carriers is either stationary or has a rotation axis stationary in space, and let us now assume that this mechanism has m ($m=1, 2, 3, \ldots$) degrees of freedom.

We shall assume that external forces are applied only to those links of mechanism K, whose rotation axes are stationary in space. One of these links, namely, the one to which the moment from the engine is applied, we shall name the driving link of the mechanism. The rest of the links, which receive externally applied moments (with the exception of the stationary link) will be termed driven links.

We shall assume that the mechanism is determinable statically, *i. e.* its conditions of equilibrium afford the possibility for the moment transmitted by any one of the links to be determined from the moment externally applied to the driving link.

Since mechanism K is statically determinable, it follows that the number of driven links in it equals m and that the angular velocities of these driven links are linearly independent of one another.

In determining the efficiency of mechanism K we shall assume that K is in equilibrium, and that the angular velocities of the driving and driven links are constant; as to the losses due to friction, we shall assume that Brandenburger's assumptions (2) hold good (see also (1)).

The symbols to be used in this paper are as follows: 0, the driving, and $1, 2, \ldots, m$, the driven links of the mechanism; ω_p, the angular velocity of link p ($p=0, 1, 2, \ldots, m$); $M_0, M_1, M_2, \ldots, M_m$, the moments transmitted, respectively, to the links $0, 1, 2, \ldots, m$, which satisfy the conditions of equilibrium when friction is not taken into account; $\tilde{M}_1, \tilde{M}_2, \ldots, \tilde{M}_m$, the corresponding moments which satisfy the conditions of equilibrium, when friction is taken into account; $i_{0p} = -M_p/M_0$, the transmission ratio between links 0 and p, set up by mechanism K when all the driven links, except link p ($p=1, 2, \ldots, m$), are stopped; $\tilde{i}_{0p} = -\tilde{M}_p/M_0$, the new value of the corresponding transmission ratio; η, the efficiency of mechanism K; η_r and i_r, the efficiency and transmission ratio, respectively, of mechanism K_r ($r=1, 2, 3, \ldots, n$) with the planet carrier stopped, and $\tilde{i}_r = i_r \eta_r^{x_r}$, where $x_r = \pm 1$ and $r=1, 2, \ldots, n$, the new value of the transmission ratio i_r.

Since the angular velocities $\omega_1, \omega_2, \ldots, \omega_m$ of the driven links 1, 2, ..., m, are linearly independent, it follows that K sets up the following correlation between $\omega_0, \omega_1, \omega_2, \ldots, \omega_m$

$$\omega_0 = i_{01}\omega_1 + i_{02}\omega_2 + \ldots + i_{0m}\omega_m \tag{1}$$

The coefficients of this correlation are the values: $i_{01}, i_{02}, \ldots i_{0m}$; they can all be expressed easily through the transmission ratios $i_1, i_2, \ldots, i_n$. We write these expressions as follows:

$$i_{0p} = f_p(i_1, i_2, \ldots, i_n), \quad p = 1, 2, \ldots, m \tag{2}$$

Using equation (2) *the efficiency of mechanism K can be calculated in the following manner:*

1. *The angular velocity* ω_0 *of the driving link of mechanism K is expressed through the transmission ratios* $i_1, i_2, \ldots, i_n$ *of the separate gear pairs which make up the mechanism, and through the angular velocities* $\omega_1, \omega_2, \ldots, \omega_m$ *of the driven links of the mechanism; this expression has the following form*

$$\omega_0 = i_{01}\omega_1 + i_{02}\omega_2 + \ldots + i_{0m}\omega_m$$

or, in virtue of (2)

$$\omega_0 = \sum_{p=1}^{m} f_p(i_1, i_2, \ldots, i_n)\,\omega_p \tag{3}$$

2. *From the formulae*

$$x_1 = \operatorname{sgn}\frac{i_1}{\omega_0}\frac{\partial\omega_0}{\partial i_1}, \quad x_2 = \operatorname{sgn}\frac{i_2}{\omega_0}\frac{\partial\omega_0}{\partial i_2}, \ldots, \quad x_n = \operatorname{sgn}\frac{i_n}{\omega_0}\frac{\partial\omega_0}{\partial i_n} \tag{4}$$

we determine the values of the indices $x_1 = \pm 1, x_2 = \pm 1, \ldots, x_n = \pm 1$.

3. *By means of the equations* $\tilde{i}_r = i_r \eta_r^{x_r}$ *we calculate the values of the new transmission ratios:* $i_1, i_2, \ldots, i_n$.

4. *Substituting in* (3) *the new values* $i_1, i_2, \ldots, i_n$ *instead of* $\tilde{i}_1, \tilde{i}_2, \ldots, \tilde{i}_n$, *we find the new values for the angular velocity* $\tilde{\omega}_0$ *of the driving link*

$$\tilde{\omega}_0 = \sum_{p=1}^{m} f_p(\tilde{i}_1, \tilde{i}_2, \ldots, \tilde{i}_n)\,\omega_p \tag{5}$$

5. *The efficiency of the mechanism is determined from the formula*

$$\eta = \tilde{\omega}_0/\omega_0 \tag{6}$$

As proof of the foregoing, note that

$$\eta = -\frac{\omega_1\tilde{M}_1 + \omega_2\tilde{M}_2 + \ldots + \omega_m\tilde{M}_m}{\omega_0 M_0}$$

Whence, applying the equation $\tilde{M}_p = -\tilde{i}_{0p}M_0$ we get

$$\eta = \frac{\tilde{i}_{0_1}\omega_1 + \tilde{i}_{02}\omega_2 + \ldots + \tilde{i}_{0m}\omega_m}{\omega_m} \tag{7}$$

The values of $\tilde{i}_{0p} = -\tilde{M}_p/M_0$ are in proportion with the moments $\tilde{M}_1, \tilde{M}_2, \ldots, \tilde{M}_m$, and they should be determined by making use of the conditions of equilibrium of mechanism K.

If we consider the complete system of equilibrium conditions for mechanism K when friction is not taken into account (let this system be designated by the symbol S)*, we find that the coefficients of this system depend only upon the values of $i_1, i_2, \ldots, i_n$. From the assumptions made at the beginning of this paper (statistical computability of mechanism K) it follows that system S affords the possibility of expressing any of the ratios $i_{01} = -M_1/M_0$, $i_{02} = -M_2/M_0, \ldots, i_{0m} = -M_m/M_0$ through the coefficients of S and, hence, through the values $i_1, i_2, \ldots, i_n$; it should be noted that the expressions for $i_{01}, i_{02}, \ldots, i_{0m}$ in terms of $i_1, i_2, \ldots, i_n$ which we obtain by employing system S, should, evidently, coincide with expressions (2), obtained previously from considerations of a kinematical nature.

If we consider the system of equilibrium conditions for mechanism K, when friction is taken into account (let this system be designated by the symbol $\tilde{S}$) we find that the coefficients of this system can be found from the corresponding ones of system S, by replacing the values $i_1, i_2, \ldots, i_n$ by $\tilde{i}_1, \tilde{i}_2, \ldots, \tilde{i}_n$. Hence we conclude that *each of the ratios* $\tilde{i}_{0p} = -\tilde{M}_p/M_0$ can be expressed in terms of $\tilde{i}_1, \tilde{i}_2, \ldots \tilde{i}_n$ just as $i_{0p} = -M_p/M_0$ can be expressed in terms of $i_1, i_2, \ldots i_n$; making use of this, we get from formulae (2)

$$\tilde{i}_{0p} = f_p(\tilde{i}_1, \tilde{i}_2, \ldots, \tilde{i}_n), \quad p = 1, 2, \ldots, m \tag{2'}$$

By means of formulae (2′) transition from equations (3) and (7) to equations (5) and (6) can be effected.

To prove formulae (4) let us consider the losses due to friction in one of the elementary mechanisms $K_1, K_2, \ldots, K_n$.

Let the mechanism K_r, in its relative motion with respect to the planet carrier, have power $\tilde{N}_r$ transmitted to it by one of its links; then the losses due to friction in the mechanism K_r can be determined from the formula

$$\Delta_r = \tilde{N}_r(1 - \eta_r^{x_r}), \quad \text{where} \quad x_r = \operatorname{sgn} \tilde{N}_r. \tag{8}$$

On the other hand, we have

$$\eta = \frac{\tilde{\omega}_0}{\omega_0} = 1 - \frac{\Delta_1 + \Delta_2 + \ldots + \Delta_n}{M_0\omega_0}$$

whence, applying (8), it follows that

$$\frac{\tilde{\omega}_0}{\omega_0} = 1 - \frac{\tilde{N}_1(1 - \eta_1^{x_1}) + N_2(1 - \eta_2^{x_2}) + \ldots + \tilde{N}_n(1 - \eta_n^{x_n})}{M_0\omega_0}$$

Differentiating both sides of this latter equation with respect to η_1 and then assuming that $\eta_1 = \eta_2 = \ldots \eta_n = 1$, we get, after cancelling x

$$\frac{i_1}{\omega_0}\frac{\partial\omega_0}{\partial i_1} = \frac{N_1}{M_0\omega_0} \text{ **}$$

where N_1 denotes the value of N_1 for $\eta_1 = \eta_2 = \ldots = \eta_n = 1$. Assuming the power $M_0\omega_0$ conveyed to mechanism K by the driving link to be positive in sign, and that $\eta_1, \eta_2, \ldots, \eta_n$ are so close to unity that $\operatorname{sgn} \tilde{N}_1 = \operatorname{sgn} N_1$ we get finally (see (8)):

$$x_1 = \operatorname{sgn} \tilde{N}_1 = \operatorname{sgn} N_1 = \operatorname{sgn} \frac{i_1}{\omega_0}\frac{\partial\omega_0}{\partial i_1}$$

and, analogously,

$$x_r = \operatorname{sgn} \frac{i_r}{\omega_0}\frac{\partial\omega_0}{\partial i_r}, \quad r = 1, 2, \ldots, n$$

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* Systems S and $\tilde{S}$ are given in [1].

** Of course, this equation could have been established without using friction [see [3]].

As an illustration of the rule for finding the efficiency let us consider, for example, the motion of an automobile along the circumference of a circle with radius R * and calculate the efficiency of its differenital gearing.

Let B be the gauge of the automobile; ω_r and ω_l the angular velocities of its right and left semi-axes; and ω_0, the angular velocity of the gear-box.

We have:

$$\omega_0 = -\frac{i_1}{1-i_1}\omega_r + \frac{1}{1-i_1}\omega_l \qquad i_1 = -1$$

$$\tilde{\omega}_0 = -\frac{\tilde{i}_1}{1-\tilde{i}_1}\omega_r + \frac{1}{1-\tilde{i}_1}\omega_l \qquad \tilde{i}_1 = i_1\eta_1^{x_1}$$

$$x_1 = \operatorname{sgn}\frac{i_1}{1-i_1}\left(1-\frac{\omega_r}{\omega_0}\right)$$

Applying the ratios $\omega_r/\omega_0 = R - {}^1/_2 B/R$ and $\omega_l/\omega_0 = R + {}^1/_2 B/R$ and assuming that $\eta = 0.94$, we find that $x_1 = 1$; $\tilde{i}_1 = -1.06$ and $\eta = \tilde{\omega}_0/\omega_0 = \frac{1}{2.06}\left(2.06 - 0.03\frac{B}{R}\right)$; and the final answer is

$$\eta = \tilde{\omega}_0/\omega_0 = 1 - 0.015\frac{B}{R}.$$

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* The radius is here taken with reference to the centre of the automobile.

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AERODYNAMICS

THE PHYSICS OF TURBULENT FLOWS

DIFFERENTIAL EQUATIONS FOR THE MOTION OF STEADY-TURBULENCE FLOWS. SIMILARITY

By V. N. VORONIN

(Communicated by A. A. Skotchinsky, Member of the Academy, 15. VIII. 1944)

I. Notation

V, $\bar{u}$, $\bar{v}$, $\bar{w}$, total average velocity of the flow at a certain point, and its components along the Cartesian coordinate axes; $\bar{U}$, $\bar{u}_x$, $\bar{u}_y$, $\bar{u}_z$, turbulent average velocity of the flow, and its components along the coordinate axes; $\bar{v}_x$, $\bar{v}_y$, $\bar{v}_z$, averaged components of the boundary velocity of the flow; $\bar{\tau}_{xy}$, $\bar{\tau}_{yz}$, $\bar{\tau}_{xz}$, $\bar{\tau}_{xx}$, $\bar{\tau}_{yy}$, $\bar{\tau}_{zz}$, averaged turbulent stresses, tangential and normal; $\bar{s}_{xy}$, $\bar{s}_{yz}$, $\bar{s}_{xz}$, $\bar{s}_{xx}$, $\bar{s}_{yy}$, $\bar{s}_{zz}$, averaged molecular stresses, tangential and normal; f, scale of turbulence; α, a static constant, preserved for all flows with steady isotropic turbulence ($\alpha \cong 0.0034$); S, dynamic cross-section of the flow; R, dynamic dimensions of the flow; P, perimeter of the dynamic cross-section of the flow; μ, molecular viscosity of the fluid; ρ, density of the fluid; p_r, hydrostatic pressure; G_x, G_y, G_z, components of mass forces.

II. Differential Equation of Turbulent Motion

1. The various turbulent flows can be boiled down to the three following classes:

a) Flows with steady turbulence. These flows are characterized by the presence of fixed boundaries, either solid or liquid, beyond which turbulence is absent unless its presence is due to some other flow. By such flows energy is continuously expended in the formation of vortical masses all along the boundary of the flow, whereby the vortex field of vortical masses is kept up in a steady state.

b) Flows with degradation of turbulence. Such flows are formed where the vortex system of vortical masses fails to be reproduced along some length of the flow, though the flow was steady up to that point, or else where the vortical masses are reproduced periodically.

c) Combined turbulent flows. These flows arise from a mixture of steady turbulent flows, or of flows with degraded turbulence, or from a mutual mixture between these two kinds.

Of the widest occurrence in Nature are flows with steady turbulence or with combined steady turbulence. In the sequel only these two classes will be discussed.

Making use of certain hypotheses on the internal structure of turbulent flows, the author has obtained the following differential equations of motion.

2. The equations of motion for an averaged field

$$\frac{\partial \bar{u}}{\partial t}+\bar{u}\frac{\partial \bar{u}}{\partial x}+\bar{v}\frac{\partial \bar{u}}{\partial y}+\bar{w}\frac{\partial \bar{u}}{\partial z}=\bar{G}_x+\frac{1}{\rho}\left[\frac{\partial(\bar{\tau}_{xx}+\bar{s}_{xx})}{\partial x}+\frac{\partial(\bar{\tau}_{yx}+\bar{s}_{yx})}{\partial y}+\frac{\partial(\bar{\tau}_{zx}+\bar{s}_{zx})}{\partial z}\right]$$

$$\frac{\partial \bar{v}}{\partial t}+\bar{u}\frac{\partial \bar{v}}{\partial x}+\bar{v}\frac{\partial \bar{v}}{\partial y}+\bar{w}\frac{\partial \bar{v}}{\partial z}=\bar{G}_y+\frac{1}{\rho}\left[\frac{\partial(\bar{\tau}_{xy}+\bar{s}_{xy})}{\partial x}+\frac{\partial(\bar{\tau}_{yy}+\bar{s}_{yy})}{\partial y}+\frac{\partial(\bar{\tau}_{zy}+\bar{s}_{zy})}{\partial z}\right]$$

$$\frac{\partial \bar{w}}{\partial t}+\bar{u}\frac{\partial \bar{w}}{\partial x}+\bar{v}\frac{\partial \bar{w}}{\partial y}+\bar{w}\frac{\partial \bar{w}}{\partial z}=\bar{G}_z+\frac{1}{\rho}\left[\frac{\partial(\bar{\tau}_{xz}+\bar{s}_{xz})}{\partial x}+\frac{\partial(\bar{\tau}_{yz}+\bar{s}_{yz})}{\partial y}+\frac{\partial(\bar{\tau}_{zz}+\bar{s}_{zz})}{\partial z}\right]$$

where

$$\bar{\tau}_{xy}=\bar{\tau}_{yx}=\bar{\rho}\bar{f}\bar{U}\left[\frac{\partial \bar{u}_x}{\partial y}+\frac{\partial \bar{u}_y}{\partial x}\right]$$

$$\bar{\tau}_{yz}=\bar{\tau}_{zy}=\bar{\rho}\bar{f}\bar{U}\left[\frac{\partial \bar{u}_y}{\partial z}+\frac{\partial \bar{u}_z}{\partial y}\right]$$

$$\bar{\tau}_{zx}=\bar{\tau}_{xz}=\bar{\rho}\bar{f}\bar{U}\left[\frac{\partial \bar{u}_x}{\partial z}+\frac{\partial \bar{u}_z}{\partial x}\right]$$

$$\bar{\tau}_{xx}=-\bar{p}-\frac{2}{3}\bar{\rho}\bar{f}\bar{U}\left[\frac{\partial \bar{u}_x}{\partial x}+\frac{\partial \bar{u}_y}{\partial y}+\frac{\partial \bar{u}_z}{\partial z}\right]+2\bar{\rho}\bar{f}\bar{U}\frac{\partial \bar{u}_x}{\partial x}$$

$$\bar{\tau}_{yy}=-p-\frac{2}{3}\rho fU\left[\frac{\partial \bar{u}_x}{\partial x}+\frac{\partial \bar{u}_y}{\partial y}+\frac{\partial \bar{u}_z}{\partial z}\right]+2\bar{\rho}\bar{f}\bar{U}\frac{\partial \bar{u}_y}{\partial y}$$

$$\bar{\tau}_{zz}=-p-\frac{2}{3}\rho fU\left[\frac{\partial \bar{u}_x}{\partial x}+\frac{\partial \bar{u}_y}{\partial y}+\frac{\partial \bar{u}_z}{\partial z}\right]+2\bar{\rho}\bar{f}\bar{U}\frac{\partial \bar{u}_z}{\partial z}$$

$$u=u_x+\bar{v}_x;\quad v=u_y+\bar{v}_y;\quad w=u_z+\bar{v}_z$$

$\bar{s}_{xy}, \bar{s}_y, \ldots, \bar{s}_{xx}$ and so on are determined in the usual way.

3. The scale of turbulence characterizes the relative size of the active portions of vortical masses in a given dynamic section of the flow, and is equal to

$$l=\alpha R=\alpha\frac{4S}{\chi}\qquad(\alpha\cong 0.0034)$$

by a dynamic section of a turbulent flow corresponding to a given point is to be understood the least among the cross-sections containing this point.

4. The field of total velocities is supposed to consist of two velocity fields — the boundary field and the turbulent field. The boundary field exists only in flows with solid boundaries (including also the laminar boundary sublayer) and is absent in flows with fluid boundaries (flows with free turbulence). In the latter case the following relations are to be added to the equations of motion referred to above

$$\bar{u}=\bar{u}_x;\quad \bar{v}=u_y;\quad \bar{w}=u_z$$

To those differential equations of motion we have to add the well-known continuity and fluid state equations and also equations expressing μ as a function of some or other parameters.

III. Boundary Conditions

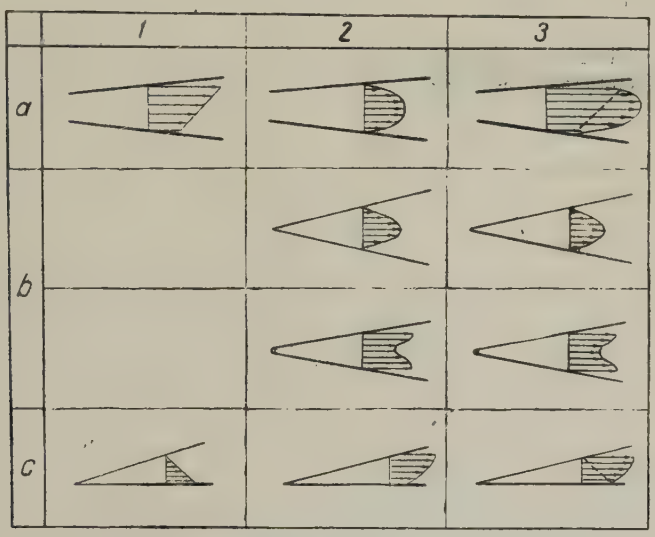
A turbulent flow is separated by its boundaries from those portions of the fluid where turbulence is absent or has a scale different from that of the flow.

If the flow is moving between solid surfaces the turbulence boundary coincides with the surface separating the laminar sublayer from the turbulent flow. Owing to this the velocity at the boundary of a turbulent flow moving between solid walls is a finite quantity, equal to the velocity of the boundary field (Fig., *a*).

If the flow is moving between fluid surfaces (free turbulence), the velocity at its boundaries is equal to that of the fluid outside the boundaries, and, at the same time, equal to the velocity of the turbulent flow at the boundaries (Fig., *b*).

For turbulent flows having both fluid and solid boundaries (Fig., *c*) the boundary conditions are found separately for each type of boundary, taking all that was said above into account.

On free surfaces of a fluid moving with steady turbulence the velocity of the



1, approximate distribution of average velocities in the boundary field; *2*, approximate distribution of average velocities in the turbulent field; *3*, approximate distribution of total averaged velocities in the flow. *a*, plane flow between solid boundaries of different roughness; *b*, flows between fluid boundaries (free turbulence), free jet and in wake of body; *c*, flow between fluid and solid boundaries, flow moving past a flat plate

flow is equal to that of the boundary field, and the turbulent velocity is equal to zero.

IV. Similarity between Fluid-Dynamic Processes in Flows of Steady Turbulence

From the differential equations of motion referred to here some new criteria expressing similarity between averaged turbulent and pulsation fields in flows of steady turbulence can be derived alongside the generally known criteria.

To honour the name of a distinguished Russian scientist, A. A. Skotchinsky, who has made profound inquiries into the complicated problems of fluid dynamics bearing on mining technology, we should like to name after him the following two criteria:

a) The first criterion of Skotchinsky, expressing similarity between averaged pulsation fields of flows, is

$$S'_k = l/f$$

b) The second criterion of Skotchinsky, expressing similarity between averaged turbulent fields, is

$$S''_k = V/U$$

Modeling of fluid-dynamics processes, in which the main part is played by turbulence, should be carried out by means of these criteria which in these cases define similarity.

With the aid of the differential equations referred to in this paper the author solved different problems connected with the velocity distribution in steady turbulence flows moving between two parallel planes, in a circular tube, in a plane-free jet, in a free jet with axial symmetry, in a plane and axially symmetrical wake of a body and also in flows with a free surface. He found his theoretical results to be in good agreement with the experimental data available in literature.

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HYDROMECHANICS

UNIVERSAL VARIABLES IN THE BOUNDARY LAYER THEORY

By A. M. FEINSILBER

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In the present paper variables of a special kind are suggested, which can as well be applied to an asymptotic boundary layer as to one of finite thickness, to a compressible as well as incompressible fluid, and are, therefore, described as universal. Their introduction simplifies the differential equations of the boundary layer, and, what is more, removes the difficulties connected with the presence of a boundary condition on the external border of the layer.

Let us adopt the following notation: u and v, the velocity components in the layer; $\bar{u}$, the value of u on the boundary of the layer; ρ, density; μ and $\nu = \mu/\rho$, the dynamic and kinematic coefficients of viscosity, respectively; τ, friction stress; x_0, $\bar{u}_0$, values at some characteristic point of the profile (*e. g.*, at the point of minimum pressure); $z = 4/3\,(u/u_0)^3$; $\mathrm{Re}_0 = \bar{u}_0 x_0/\nu$; $T_0 = \rho \bar{u}_0^2/2$.

The differential equations of a laminar boundary layer are [1]

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \rho \bar{u}\bar{u}' + \frac{\partial \tau}{\partial y} \qquad (1)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (2)$$

$$\tau = \mu \frac{\partial u}{\partial y} \qquad (3)$$

Introduce dimensionless independent variables $x_1 = x/x_0$; $u_1 = u/\bar{u}_1$ and a dimensionless function

$$\tau_1 = \frac{\sqrt{\mathrm{Re}_0}}{T_0}\,\tau \qquad (4)$$

In new variables the system (1)—(3) is reduced to the equation

$$\tau_1^\circ \frac{\partial^2 \tau_1}{\partial u_1^2} - z'(1 - u_1^\circ)\frac{\partial \tau_1}{\partial u_1} - 3z u_1 \frac{\partial \tau_1}{\partial x_1} = 0 \qquad (5)$$

To begin with, we shall consider the most important particular cases.

1. Plate. In this case $\bar{u} = \bar{u}_0$; $u_1 = f(t)$, where $t = \sqrt{\bar{u}_0/\nu x}\cdot y$.

Putting

$$\alpha = \frac{\sqrt{x_1}}{2} \quad \tau_1 = \frac{du_1}{a} \qquad (6)$$

where α is a function of u_1 only, we obtain an ordinary differential equation

$$\alpha \frac{d\alpha}{du_1^2} = -\frac{u_1}{2} \qquad (7)$$

We look for a solution in the form of a series

$$\alpha = \sum_{i=0}^{\infty} a_i u_1^i$$

For the coefficients a_i the following theorems can be established:

Theorem 1. *If* $i \geqslant 1$, *then* $a_i \leqslant 0$.

Theorem 2. *If* i *is no multiple of* 3, *then* $a_i = 0$.

The coefficients of the series are determined by a recurrent formula ($a_0 = 0.332$)

$$a_{3k} = -\frac{1}{(3k-1)\,ka_0} \sum_{p=1}^{k-1} (3p-1)\, pa_{3(k-p)} a_{3p} \qquad (9)$$

The solution has the form

$$\alpha = 0.332 - 0.251u_1^3 - 0.038u_1^6 - 0.014u_1^9 - \\ - 0.007u_1^{12} - 0.004u_1^{15} - 0.003u_1^{18} - 0.002u_1^{21} - \ldots$$

The velocity distribution is determined from (6), (7) and (8). So we have

$$\sqrt{\frac{\overline{u}_0}{\nu x}}\, y = t = -2 \int_0^{u_1} \frac{d\alpha}{du_1^2}\, du_1 = -6 \sum_{k=1}^{\infty} \frac{k(3k-1)}{3k-2}\, a_{3k} u_1^{3k-2}$$

2. Power-distribution of velocities $\overline{u} = (ax_1 + b)^n$(2). In this case $u_1 = f(\theta)$, where $\theta = yu^{\frac{n-1}{n}}$. Putting $\alpha = u^{\frac{1-3n}{2n}} \tau_1$ we obtain, seeing that α is a function of u_1 only, an ordinary differential equation

$$\alpha^2 \frac{d^2\alpha}{du_1^2} + n(u_1^2 - 1) \frac{d\alpha}{du_1} - \frac{3n-1}{2} u_1 \alpha = 0 \qquad (10)$$

In the particular case of a diffusor ($n = -1$) this equation is reduced to quadratures. In the general case the solution is again thought in the form of the series (8). The coefficients of the series are determined by a recurrent formula similar to (9). The solution has the form

$$\alpha = a^0 - \frac{n}{a_0} u_1 - \frac{n^2}{2a_0^3} u_1^2 + \left(\frac{3n-1}{12a_0} - \frac{n^3}{2a_0^5}\right) u_1^3 + \\ + \left(\frac{13n^2 - 3n}{48a_0^3} - \frac{5n^4}{8a_0^7}\right) u_1^4 + \frac{1}{120}\left(\frac{49.5n^3 - 10n^2}{a_0^5} - \frac{105n^5}{a_0^9}\right) u_1^5 + \ldots \qquad (10a)$$

where $a_0(n)$ is a known coefficient.

Let us now turn to an arbitrary form of profile.

Expansion in the neighbourhood of the critical point (in the case of a symmetrical profile even powers are absent)

$$\tau_1 = \sum_{i=1}^{\infty} A_i (u_1)\, x^i \qquad (11)$$

gives a non-linear equation for determining the first coefficient ($b_1(u_1) = A_1(u_1)/\sqrt{(\partial z/x_1^3)_{x_1=0}}$)

$$b_1^2 b_1'' - (1-u_1^2)\, b_1' - u_1 b_1 = 0 \qquad (12)$$

and a series of linear equations for the subsequent coefficients

$$b_1^2 A_n'' - (1-u_1^2)\, A_n' + \left[\frac{2(1-u_1^2)\, b_1'}{b_1} + (2-n)\, u_1 \right] A_n = Q_n, \quad n \geqslant 2 \qquad (13)$$

where $Q_n(u_1)$ is a known function.

Equation (12) is a particular case of equation (10) for $n=1$; so its solution is expressed by formula (10a) for $n=1$. Equations (13) are integrated by series whose coefficients are determined by means of recurrent formulae from a linear system of algebraic equations.

The abscissa of the separation point x_s is determined by the equation

$$A_1(0) + A_2(0)\, x_s + A_3(0)\, x_s^2 + \ldots = 0$$

The velocity distribution is determined by the series

$$\frac{T_0 x_1 y}{\mu u \sqrt{\mathrm{Re}_0}} + \int_0^{u_1} \frac{du_1}{A_1(u_1)} - x_1 \int_0^{u_1} \frac{A_2(u_1)\, du_1}{A_1^2(u_1)}\, du_1 \ldots \qquad (14)$$

The universal variables suggested here have also been used by the author in the case of a boundary layer of a compressible fluid.

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TECHNICAL PHYSICS

THE GENERAL RECIPROCITY THEOREM IN THE THEORY OF RECEIVING AND TRANSMITTING ANTENNAE

By J. N. FELD

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The reciprocity theorem for two arbitrary antennae in the case of harmonic oscillations of one and the same frequency f is written as follows[1 *]

$$I^{(1)}\mathfrak{E}_1 = I^{(2)}\mathfrak{E}_2 \tag{1}$$

where $I^{(1)}$ and $I^{(2)}$ are the currents passing through the terminals of the first and second antennae when they are operating as receiving aerials, and $\mathfrak{E}_1$ and $\mathfrak{E}_2$ are the total emf's of the generators connected to the terminals of the antennae when the latter are operating as transmitting aerials. Moreover, when one of the antennae is a transmitting aerial, the other functions as a receiving aerial, and *vice versa*.

Theorem (1) holds, however, only under the condition that the internal (complex) resistances of the generators z_1 and z_2 corresponding to the resistances of the receivers $z^{(1)}$ and $z^{(2)}$ connected to the terminals of the antennae when the latter are functioning as receiving aerials, are equal. In the general case, when this condition is not fulfilled, theorem (1) should be replaced by a more general one, whose derivation is considered here.

Let us consider first of all a method of taking into account the generators and receivers connected to the terminals of the antennae. The simplest way of doing this is to replace an actual system consisting of an antenna with a generator or a receiver by one antenna on which an external emf has been applied to the section between the terminals *, so chosen that the distribution of the currents and charges in the antenna does not vary.

In the case of transmission, this emf should be equal to the potential difference developed by the generator attached to the terminals of the antenna, *i. e.*

$$\left.\begin{aligned}\mathfrak{E}_1^{\text{ext}} &= \mathfrak{E}_1 - I_1 z_1 = I_1 z_{a1} \\ \mathfrak{E}_2^{\text{ext}} &= \mathfrak{E}_2 - I_2 z_2 = I_2 z_{a2}\end{aligned}\right\} \tag{2}$$

Here I_1 and I_2 are the currents flowing through the terminals of the first and second antennae, respectively, when the latter are functioning as transmitting aerials, and z_{a1} and z_{a2} are the resistances of the antennae referred to their terminals. The other notations are the same as above.

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* Here the terminals should be regarded as short circuited by an ideal linear conductor along which the outside emf is impressed.

For the receiving antenna the external emf is evidently equal to the fall in potential in the receiver with the opposite sign

$$\left.\begin{aligned} \mathfrak{E}^{(1)\,\mathrm{ext}} &= -I^{(1)}z^{(1)} \\ \mathfrak{E}^{(2)\,\mathrm{ext}} &= -I^{(2)}z^{(2)} \end{aligned}\right\} \tag{3}$$

Thus by introducing a fictitious external emf equivalent to the influence of the receiver we formally transform the receiving aerial into a transmitting aerial.

Let $\vec{E}'$, $\vec{H}'$ denote the field set up by the antennae when an external emf $\mathfrak{E}_1^{\mathrm{ext}}$ is applied to the first of them (transmitting) and an external emf $\mathfrak{E}^{(2)\mathrm{ext}}$ to the second (receiving), and let $\vec{E}''$ $\vec{H}''$, denote the field set up by the antennae when an external emf $\mathfrak{E}^{(1)\mathrm{ext}}$ is applied to the first of them (receiving) and an external emf $\mathfrak{E}^{\mathrm{ext}}$ to the second (transmitting).

Except at the points where the external emf's are applied *, both introduced fields everywhere satisfy the Lorentz (4) lemma

$$\operatorname{div}[\vec{E}'\vec{H}''] = \operatorname{div}[\vec{E}''\vec{H}'] \tag{4}$$

In our case the external emf's are applied to the surfaces s_1 and s_2 of ideal conductors that short-circuit the terminals of the first and the second antenna (see footnote on page 476). Therefore upon integrating the equality (4) over the infinite space bounded from within by the surfaces s_1 and s_2 and then applying the Gauss theorem, we get

$$\int\limits_{s_1+s_2} [\vec{E}'\vec{H}'']\vec{ds} = \int\limits_{s_1+s_2} [\vec{E}''\vec{H}']\vec{ds} \tag{5}$$

Let us apply the boundary conditions for the tangential components of the field vectors at the surface of ideal conductors when there are external emf's impressed upon them. In our case these conditions have the form

$$\left.\begin{aligned} E_t' &= -E_t'^{\mathrm{ext}}[\vec{n}\vec{H}'] = \frac{4\pi}{c}\vec{i}' \\ E_t'' &= -E_t''^{\mathrm{ext}}[\vec{n}\vec{H}''] = \frac{4\pi}{c}\vec{i}'' \end{aligned}\right\} \begin{array}{l}\text{on } s_1 \\ \text{and} \\ s_2\end{array} \tag{6}$$

where $\vec{i}'$ and $\vec{i}''$ are the surface current densities corresponding to the fields $\vec{E}'$, $\vec{H}'$ and $\vec{E}''$, $\vec{H}''$; $\vec{n}$ is the unit vector, normal to the corresponding surface; $\vec{E}'^{\mathrm{ext}}$ and $\vec{E}''^{\mathrm{ext}}$ are the potenitals of the external emf's (exciting the fields $\vec{E}'$, $\vec{H}'$ and $\vec{E}''$ $\vec{H}''$), connected with the corresponding external emf's by the obvious equalities **:

$$\left.\begin{aligned} \mathfrak{E}_1^{\mathrm{ext}} &= \int\limits_{(l_1)} \vec{E}'^{\mathrm{ext}}\,\vec{dl};\quad \mathfrak{E}^{(2)\mathrm{ext}} = \int\limits_{(l_2)} \vec{E}'^{\mathrm{ext}}\,\vec{dl}; \\ \mathfrak{E}^{(1)\mathrm{ext}} &= \int\limits_{(l_1)} \vec{E}''^{\mathrm{ext}}\,\vec{dl};\quad \mathfrak{E}^{\mathrm{ext}} = \int\limits_{(l_2)} \vec{E}''^{\mathrm{ext}}\,\vec{dl} \end{aligned}\right\} \tag{7}$$

Integration in formulae (7) is carried out along the length l_1 and $_2$ of the linear sections of the first and second antennae to which the external emf's are applied (see footnote on page 476).

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* And also at the surfaces of separation between the two media, which, however, is not essential in what follows, since the quantity $[EH]_n$ remains on them continuous.

** The law of the distribution of the external emf's has no effect here, since they are impressed along linear conductors which are small in comparison with the wave length.

Substituting the values of E_t and H_t from formulae (6) into equality (5), we find after elementary rearrangement

$$\int\limits_{(s_1)} (\vec{i''}\vec{E'}^{\text{ext}} - \vec{i'}\vec{E''}^{\text{ext}})\, ds = \int\limits_{(s_2)} (\vec{i'}\vec{E''}^{\text{ext}} - \vec{i''}\vec{E'}^{\text{ext}})\, ds. \tag{8}$$

The vectors $\vec{i'}$, $\vec{i''}$ do not vary along the lengths l_1 and l_2 of the linear conductors *, coincide in direction with their axes and are connected with the total currents passing through the terminals of the antennae by the obvious equalities

$$\left.\begin{aligned} I_1 &= \oint\limits_{(L_1)} i'dl; \quad I^{(2)} = \oint\limits_{(L_2)} i'dl \\ I^{(1)} &= \int\limits_{(L_1)} i''dl; \quad I_2 = \int\limits_{(L_2)} i''dl \end{aligned}\right\} \tag{9}$$

Here L_1 and L_2 are the contours of the cross-section of the conductors short-circuiting the terminals of the first and second antennae.

Taking into account formulae (7) and (9), we may write equation (8) in the following form

$$I^{(1)}\mathfrak{E}_1^{\text{ext}} - I_1\mathfrak{E}^{(1)\text{ext}} = I^{(2)}\mathfrak{E}^{\text{ext}} - I_2\mathfrak{E}^{(2)\text{ext}} \tag{10}$$

The equality thus obtained is the general reciprocity theorem for receiving and transmitting antennae. The external emf's that enter into it are defined by formulae (2) and (3) and have a simple physical meaning. By making use of formulae (2) and (3), theorem (10) may also be written in the following two equivalent forms

$$I^{(1)}I_1(z_{a1} + z^{(1)}) = I^{(2)}I_2(z_{a2} + z^{(2)}) \tag{11}$$

and

$$I^{(1)}\mathfrak{E}_1\frac{z_{a1} + z^{(1)}}{z_{a1} + z_1} = I^{(2)}\mathfrak{E}_2\frac{z_{a2} + z^{(2)}}{z_{a2} + z_2} \tag{12}$$

From the last expression, by the way, it is evident that in the particular case where $z_1 = z^{(1)}$ and $z_2 = z^{(2)}$ the general reciprocity theorem passes over into the familiar theorem (1).

If we introduce, as is usually done in the theory of receiving antennae, the conception of the total emf of the receiving antenna with the aid of formulae

$$\mathfrak{E}^{(1)} = I^{(1)}(z_{a1} + z^{(1)})$$

and

$$\mathfrak{E}^{(2)} = I^{(2)}(z_{a2} + z^{(2)})$$

then equality (11) may be written in the particularly compact form

$$I_1\mathfrak{E}^{(1)} = I_2\mathfrak{E}^{(2)} \tag{13}$$

This form of the general theorem is very much like theorem (1), but, as can be seen from the foregoing, is not equivalent to it.

By a method similar to the one employed here it is also possible to generalize the reciprocity theorem ([5]) for non-sinusoidal oscillations.

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* Since l_1 and l_2 are many times smaller than the wave length.

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BIOPHYSICS

SENSATION ELEMENTS FOR HEARING AND TOUCH

By S. J. LIFSHITZ

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In an earlier paper (1) the author described a new phenomenon—«fluctuation of the hearing threshold». Next (2) he was able to show that a similar phenomenon—«fluctuation of the tactile threshold»—might be established qualitatively and quantitatively in relation to the sense of touch.

When the intensities of sound were near to hearing threshold (2 db), some of a number of equal signals began to disappear statistically from perception. The probability of a signal thus dropping out was found to be a function of the intensity of sound.

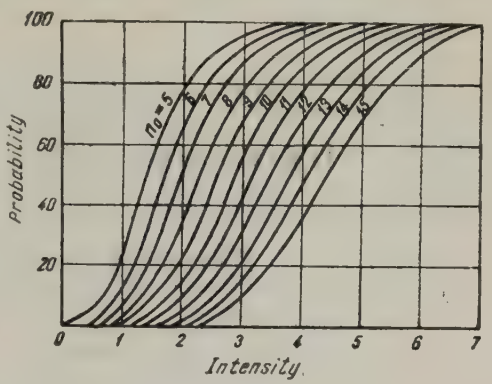

Fig. 1

It was also shown (2) that man's perceptions are discrete, composed of separate elements. Assuming that n_0 elements are required to attain the perception threshold, the theory of chances can be employed to determine this number from the data of an experiment in which the probability of appearance or disappearance of the signal is dealt with as a function of sound intensity.

In treating the experimental material Poisson's law of statistic distribution may be applied

$$P_z = \frac{n^z e^{-n}}{z!}$$

Here P_z denotes the probability of the sound impulse just creating z elements. Sound feeling occurs when the acoustic nerve fibre perceives n_0 elements or more.

The probability that the sound impulse will create n_0 elements and more, up to infinity, is determined as a sum of probabilities.

$$'^{\infty}\Pi'_{n_0} = e^{-n} \sum_{z=n_0}^{\infty} \frac{n^z}{z!}$$

The solution of this equation gives a family of Poisson's curves for different values of n_0 (Fig. 1).

The probability of an auditory or tactile effect is determined by experiment for sound impulses of different intensity. In this way a number of experimental

Table 1

Fluctuation of Hearing Threshold

Frequency	1000 cycles			250 cycles
Observer	N. B.	B. G.	G. A.	N. B.
n_0	9	10	9	6
»	8	9	6	12
»	7	12	5	9
»	12	8	8	7
»	9	9	10	5
»	10	8	9	8
»	6	10	11	7
»	11	9	9	8
»	8	7	10	8
»	7	9	5	6
Mean . .	9	9	8	7

For the average observer $n_0 = 8$.

Table 2

Fluctuation of Tactile Threshold

Frequency	50 cycles				250 cycles
Observer	N. B.	G. A.	B. L.	S. L.	N. B.
n_0	9	8	5	8	7
»	8	7	7		5
»	9	6	5		5
»	8	7	10		7
»	12	5			8
»	11	10			8
»	9	7			5
»	7	8			11
»	7	10			13
»	11	9			9
Mean .	9	8	7	8	8

For the average observer $n_0 = 8$.

points (six, in our case) is obtained, and it remains to select from the family of Poisson's curves the one in which these points fit best.

The experiments were run as follows. In a deadened room* an observer at a telephone listened to sound signals sent in succession at intervals of 1.2 sec. (from end of one signal to beginning of the next), each lasting 0.3 sec. The sound impulses were created by a tube generator. In every single experiment there were

about 50 such impulses, all of the same intensity of sound. The observer recorded the number of impulses perceived by his ear. The ratio of this number to the number of the impulses sent is a measure of the probability of perception for a given intensity of sound.

Afterwards the intensity of sound was reduced by 12 per cent (0.5 dl), the probability of perception measured again, and so on. Generally, the measurements were made for probabilities ranging from 90 to 20 per cent. Then the Poisson curve was selected (Fig. 1), which in its course came nearest to the data obtained. This gave the value of n_0.

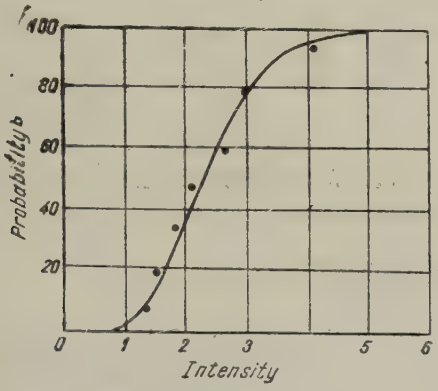

Fig. 2

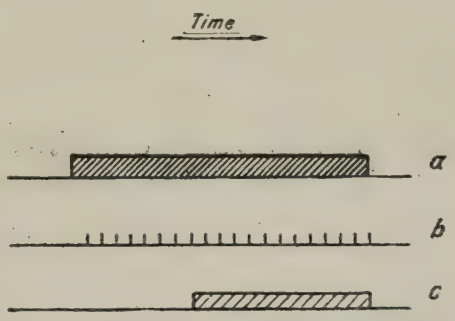

Fig. 3

In tactile threshold experiments the telephone had a membrane provided with a needle. The observer kept his finger in contact with the point of the needle and perceived the impulses sent by the generator with his sense of touch. Otherwise the procedure was the same as in the hearing threshold experiments.

Tables 1 and 2 record the values of n_0 obtained in that way. Each observer performed 10 series of observations, one series every day, and for each series the value of n_0 was determined.

Fig. 2 is annexed to exemplify one series of tests: hearing, frequency 1000 cycles.

Observer N. B.

Perception level in db	3	2.5	2	1.5	1	0.5
Intensity of sound	3	2.7	2.1	1.8	1.6	1.4
Probability	79	60	48	33	19	7

$n_0 = 8$

It follows from the experiments that eight elements are required for acoustic or tactile perception. Most likely these elements correspond to electric discharges in the nerve fibre, which, according to Adrian, are involved in every act of perception (action current impulses). Fletcher's theory of hearing, which is based on experimental lata, likewise suggests occurrence of electric discharges in the nerve fibre.

This idea agrees with the «accumulation» phenomenon discovered by the author (Fig. 3). In Fig. 3 *a* is the sound impulse, and *c* is the perceptional effect of this impulse, which does not appear until some time afterwards when there is sufficient accumulation of energy to attain the threshold of perception. Line *b* shows that the sound impulse first creates electric discharges in the nerve fibre and only after the eighth discharge does sound feeling take place. The same, of course, is true of the sense of touch.

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* Carried out in the Physical-Technological Laboratory of the Uzbekistan Branch of the Academy of Sciences of the USSR, in Tashkent.

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CHEMISTRY

PRODUCTION OF α-METHYL STYRENE BY CATALYTIC DEHYDROGENATION OF ISOPROPYL BENZENE

By A. A. BALANDIN, Corresponding Member of the Academy, and G. M. MARUKJAN

In previous communications [1, 2] it has been shown that catalytic dehydrogenation of ethyl benzene gives high yields of styrene; this process may be carried out either with or without dilution (with carbon dioxide of nitrogen), which, when used, increases the yield.

We have extended this investigation to the dehydrogenation of isopropyl benzene (cumene). The initial isopropyl benzene had the following constants: b. p. 149—152°, $d_4^0 = 0.8606$ and $n_D^{20} = 1.4897$. The experimental procedure (the flow method, atmospheric pressure), the catalyst, and the method of analysis (determination of the bromine numbers and fractional distillation in a Fenske type column) were the same as in the previous works[1,2].

Below are given the results of two series of experiments, one of them involved dilution with carbon dioxide, in a molecular ratio of $1C_9H_{12} : 2CO_2$, and the other was carried out without dilution (see table). In both series the temperature was kept constant at 625°, the velocity of flow was 450—600 ml of liquid isopropyl benzene per litre of catalyst per hour; the experiments show that variation of the velocity within these limits has little effect upon the yield. The table gives the yields and the concentrations of α-methyl styrene in the liquid condensate as a function of the time during which the catalyst was in service, showing also the effect of regenerating the catalyst. Just as in the previous works, regeneration was carried out by passing a current of air over the catalyst (the furnace being heated to the temperature of the reaction) until the oxidation of the carbonaceous films on the catalyst was accomplished, which could be observed by following the changes in temperature of the latter.

The data in the table and a comparison with those given previously suggest the following conclusions. Isopropyl benzene is more easily converted into α-methyl styrene by catalytic dehydrogenation than ethyl benzene into styrene, under equal conditions. Under the conditions described isopropyl benzene gives a high yield of α-methyl styrene. Dilution with carbon dioxide stimulates the dehydrogenation of isopropyl benzene. The activity of the catalyst, after an initial increase which is often observed with chromium catalysts, gradually decreases, but remains at a high level for about 15 hrs. Regeneration in the apparatus by passing a current of air through it restores the activity of the catalyst. Most of the α-methyl styrene can easily be distilled away from the condensate in a Fenske type column at atmospheric pressure. Using the method described about 200 g of a substance, b. p. 160—162°; d_4^{20} 0.9056; n_D^0 1.5360, was produced from isopropyl benzene; analysis (bromine addition method) proved it to contain 98 per cent α-methyl styrene.

No. of experiment	Length of service of catalyst, hrs.	Concentration of C_9H_{10} in condensate, p. c. by wt.	Yield of C_9H_{10} based on C_9H_{12} passed	No. of experiment	Length of service of catalyst, hrs.	Concentration of C_9H_{10} in condensate, p. c. by wt.	Yield of C_9H_{10} based on C_9H_{12} passed
In the presence of CO_2 (1:2) at 625°				Without dilution at 625°			
548[1]	7	65.0	59.2	599[5]	1	57.2	50.0
549	7.5	72.8	70.9	600	1.5	53.3	48.8
550	8.5	69.3	61.0	601	3	49.7	43.4
575	16.5	64.5	57.5	602	4	47.4	45.0
585[3]	8	68.6	59.1	603	7.5	44.8	38.1
586	14	70.0	61.6	604	18.5	41.9	37.3
587	18	85.2	50.5	605	16.5	43.3	38.2
588	21.5	41.8	—	606	20.5	39.7	35.3
589[3]	24	66.4	53.2	607	24	28.2	26.9
590	28	54.0	49.4	608[6]	26	47.5	—
591	30	50.0	47.0	609	31.5	37.2	34.2
592	34.5	44.9	—	610	32.5	34.1	28.1
597[4]	32.5	50.8	40.3	613[7]	36	45.6	42.8
				614	38	39.3	32.4
				615[8]	1	40.2	34.6
				616	2.5	50.1	—
				617	5	42.2	30.7
				618	9	44.6	37.0
				619	15	39.0	29.0
				620	18	39.1	22.9
				621	23	44.7	31.5
				622	24.5	35.5	25.8
				623[9]	27.5	43.5	—

[1] Duration of experiment No. 548, 30 mins
[2] New portion of catalyst.
[3] Catalyst regenerated.
[4] Catalyst regenerated and experiment No. 597 without CO_2.
[5] Experiment No. 599 carried out with CO_2 (1:2); duration, 30 min.
[6] Catalyst regenerated.
[7] Catalyst regenerated.
[8] New portion of catalyst. Experiment No. 615 carried out with ethyl benzene in presence of CO_2. Experiments Nos. 616—622, with isopropyl benzene without CO_2.
[9] Catalyst regenerated.

It is a point of theoretical interest that the introduction of a methyl radical into the ethyl benzene molecule in the α-position increases the reaction velocity of the catalytic dehydrogenation.

This work was carried out in 1942.

Institute of Organic Chemistry.
Academy of Sciences of the USSR.

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CHEMISTRY

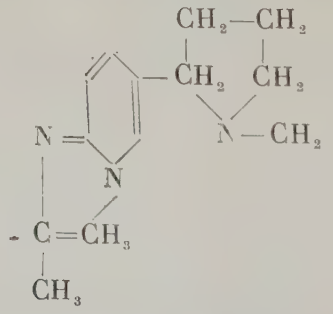
ON THE ACTION OF ALKYL HALIDES ON AMINO-DERIVATIVES OF NICOTINE

By J. L. GOLDFARB and M. S. KONDAKOVA

(Communicated by A. N. Nesmeyanov, Member of the Academy, 15. XI. 1944)

In a former paper (1) on the synthesis of condensed compounds from α-aminonicotine and α′-aminonicotine it has been mentioned that the second amine differed from the first one and when reacting with chloracetone gave but a poor yield of the corresponding substituted pyrimidazole. The main amount of α′-aminonicotine yielded in that reaction a product that could not be extracted with ether. A similar difference in the behaviour of these amines we again observed later when studying their reactions with other α-halogen-carbonyl compounds.

The formation of products insoluble in ether may, as had already been mentioned previously (1), have resulted from a molecule of chloracetone being added to the pyrrolidine nitrogen atom of either the aminonicotine molecule, or the molecule of the newly formed base. Practically, any of the three nitrogen atoms of the aminonicotine molecule can become the centre of fixation of compounds with a labile halogen atom. What is more, the pyrrolidine nitrogen atom, as a nitrogen of a «fatty» amine, is remarkable for its ability to add such halogen-containing compounds. Accordingly, if the pyrrolidine nitrogen atom of the molecule of α′-aminonicotine was brought beforehand to a saturation condition, it would be possible so to restrain the action of halogen-carbonyl compounds on the amine as to bring only the pyridine part of the molecule into the reaction. This was proved by us experimentally. Instead of the free α′-aminonicotine which, like other nicotine derivatives, is in fact a bi-acidic base, we used its monohydrochloride in the reaction with chloracetone and achieved at once a considerable increase in the yield of 5-(*N*-methyl-α-pyrrolidil)-2-methyl-pyrimidazole (I)



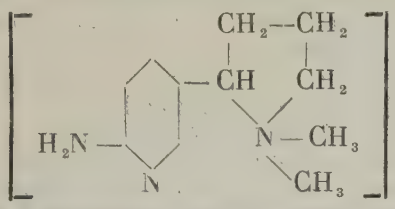
(I)

Indirect evidence was thus obtained in support of the assumption that products with addition across the pyrrolidine nitrogen atom were formed in the reaction of α-halogen-carbonyl compounds with aminonicotines and, in particular, with α′-aminonicotine. As these products had not, however, been isolated in

an individual state, many problems remained unsolved, particularly that of the difference in the behaviour of isomeric aminonicotines mentioned above. This made us take up a systematic study of the action upon aminonicotines of compounds with a labile halogen and beginning with the simplest compounds—the alkyl halides. At this juncture we are already in a position to communicate some data obtained by us, but their interpretation will be the subject of a future study.

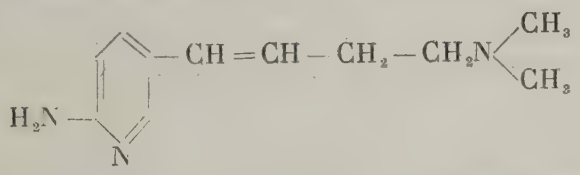
It should be noted that even during the first experiments when we acted with methyl iodide upon α- and α′-aminonicotines we again found a difference in the behaviour of these amines towards halogen compounds. After a cooled concentrated solution of α′-aminonicotine in methyl alcohol had been mixed with 1 mole of methyl iodide, and the reaction brought to its end by heating the reaction mixture for several minutes, the subsequent addition of ether to the latter resulted in the appearance of an oil readily soluble in water which did not turn into crystals for a long time.

Upon treatment of this oil with a concentrated alkali solution and shaking it with ether and benzene, only a very small quantity of the substance could be extracted. Often the oil crystallized as large cubic crystals which after two crystallizations from ethyl alcohol melted when heated slowly, at 149—150°. It should be noted, however, that the melting point depends on the rate of heating. These crystals contain crystallization water ($^1/_2$ mol.) and according to analysis they are the addition product of methyl iodide to α-aminonicotine (ratio 1 : 1). Under the action of picric acid a picrate is formed; temperature of decomposition 217°. With hydrochloric acid a hydrochloride is formed; m. p. 114°. Simply from the fact that this iodide, m. p. 149—150°, undergoes no change under the action of alkalies one feels entitled to assign it the structure (II)



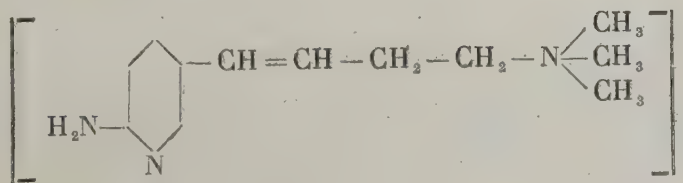
(II)

Its nature as a salt of a quaternary base is further confirmed by the study of its splitting off according to Hofmann. Distillation *in vacuo* of the base formed by treating this iodomethylate with silver oxide yields a crystalline compound, which after crystallization from a mixture of benzene with gasoline has a m. p. 96—97°. The composition of this base agrees well with the data calculated for a compound having the structure (A)



(A)

In an acid medium the base, m. p. 96—97°, immediately desoxidizes potassium permanganate and forms with methyl iodide a new iodide, m. p. 210°, which undergoes no change under the action of alkalies. From this property and from its analysis data the substance appears to be a iodomethylate, whose structure can be expressed by formula (III)

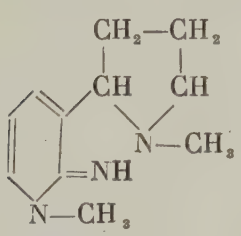


(III)

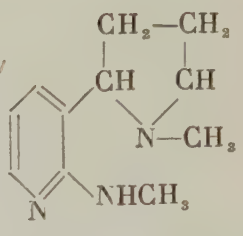
α-aminonicotine which is an isomer of α′-aminonicotine reacts with methyl iodide less rigorously. The investigation of the reaction product showed that in this case it contained besides the initial amine which had not entered into reaction, a substitution product and at least two halogen-alkylates which after crystallization from alcohol melted with decomposition at 226—228° and 205°.

The structure of the iodomethylate, m. p. 226—228°, was determined by us formerly (2). Degradation according to Hofmann yields a base, m. p. 88°. The base obtained in the methylation of α-aminonicotine is difficult to purify, and only through crystallizing its salt with chloric acid a product boiling within a narrow range (153—154° at 8 mm) is isolated which is in the main the monomethyl derivative of α-aminonicotine.

Its structure can be expressed either by formula (IVa) or (IVb)



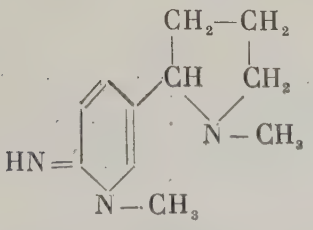
(IVa)



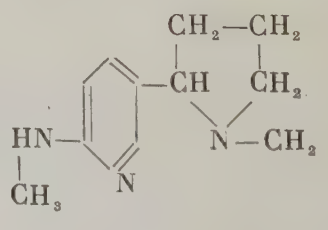
(IVb)

The fact that its vapours are coloured is in favour of structure (IVa). Nevertheless, it should be noted that when brought to boiling with an alkali, no formation of ammonia takes place, whereas the formation of ammonia under such conditions is characteristic of compounds having a structure of this kind.

The yield of the iodomethylate is about 25—28 per cent of the theoretical amount. The ratio of the substitution product formed, containing, however, some initial amine, amounts to about 50 per cent. By comparing these data with those obtained in studying the action of methyl iodide upon α′-aminonicotine (the yield of the corresponding iodomethylate, calculated for the product that has not been recrystallized, is about 65 per cent, and that of bases that can be extracted—less than 20 per cent), one arrives at the conclusion that in reaction with α-aminonicotine the bond of this iodide is established mainly, through one of the nitrogen atoms in the pyridine part of the molecule, and in the reaction with α′-aminonicotine the linkage is exclusively, or almost exclusively, through the pyrrolidine nitrogen atom. Accordingly, the following conclusion can be drawn: in the methylation of α′-aminonicotine the substitution product can be formed only under the condition that the pyrrolidine nitrogen atom of the molecule of this base is saturated beforehand. We proved this conclusion to be valid by acting with methyl iodide upon the monohydroiodide of α′-aminonicotine. Thus a methylation product was obtained having the structure (Va) or (Vb) (b. p. 165° at 11 mm)



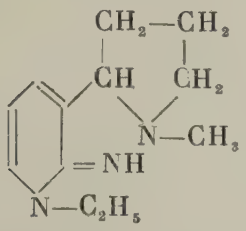
(Va)



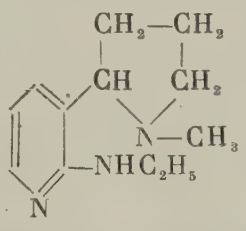
(Vb)

The remarkable property of this base is that in the air it very rapidly adds carbon dioxide, whereby a crystalline carbonate is formed. Making good use of this property, it is possible by passing a current of moist carbon dioxide through an etheral solution of α′-aminonicotine and of its methylation products to separate the latter from the initial amine. The base itself (Va) crystallizes on standing and forms crystals which deteriorate very rapidly in the air.

The conclusions we had arrived at in studying the reaction between methyl iodide and aminonicotines could not yet be used as a basis for the solution of the problem connected with the direction of the processes taking place between these bases and other alkyl halides. Among the latter methyl iodide stands out alongside of allyl iodide owing to its exceptionally high reactivity. In the case investigated this circumstance might have caused the reaction to proceed mainly in one definite direction. So we were obliged to extend the range of our experiments against the initial plan, so as to include alkylation of aminonicotines with ethyl and propyl iodides. Very soon we discovered that ethyl iodide, contrary to methyl iodide with α-aminonicotine, yields no halogen-alkylate, but only forms a substitution product—an ethylamino-derivative (b. p. 152—154° at 8 mm; picrate, m. p. 193—194°, the yield being 60 per cent). For its separation from the initial amine the mixture of the two bases is treated with nitrous acid. The ethylation product forms no nitrosoamine, this possibly being the consequence of its having structure (VIa) and not (VIb)



(VIa)



(VIb)

In the reaction with α-aminonicotine propyl iodide like ethyl iodide forms a substitution product only.

Thus we can see that the substitution of methyl iodide by a less effective alkylating agent leads to a sharp decrease in the rate of the formation of Pl-halogen-alkylates (Pl-pyrrolidine), and, probably, to an increase in the rate of alkylation of the aminopyridine part of the molecule.

A phenomenon of this kind is also observed in the reaction of ethyl and propyl iodides with α′-aminonicotine, but the decrease in the ability of the pyrrolidine nitrogen atom to form halogen-alkylates is here not so pronounced.

The data referred to above to describe differences in the reactivity of amino-derivatives of nicotine, are evidence of the rôle of the nature of the nitrogen atom as a factor determining the direction of the process which takes place when organic bases react with compounds having a labile halogen atom. From this standpoint the facts described above go beyond the range of the problems bearing on derivatives of nicotine.

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PHYSICAL CHEMISTRY

THE PHENOMENON OF CONTRACTION AND ITS APPLICATION IN CHEMICAL KINETICS

By N. M. EMANUEL

(Communicated by N. N. Semenov, Member of the Academy, 30. XI. 1944)

One of the central problems in chemical kinetics is the nature and concentrations of the intermediate products that form in the course of complex reactions.

Examining theoretically various types of chemical transformation, Semenov (1) drew the conclusion that in non-stationary branching chain reactions appreciable concentrations of active intermediate products may be built up. This has been substantiated by experiment. Thus the formation of the free radicals OH, CS and SO in appreciable concentrations has been proved in the oxidation of hydrogen, carbon disulphide and hydrogen sulphide, respectively. There have also been numerous observations of the appearance of large quantities of active products in the form of peroxides and aldehydes during the oxidation of hydrocarbons. The chain theory of chemical reactions ascribes a decisive rôle in the development of the process to free atoms and radicals. If the concentration of the radicals and atoms forming in the course of chemical transformation is great, a kinetic study of appropriate reactions may be expected to reveal a number of specific phenomena.

For example, by interrupting the process, we should observe the contraction phenomena due to the recombination of the atoms and radicals suddenly deprived of the possibility of participating in the further development of the chain reaction. Here the amount of contraction may serve as an indicant of the absolute quantities of these substances in the reaction mixture*.

The contraction curves taken as the reaction progresses should have a maximum characterizing the change in the concentration of the intermediate products.

The thermal reaction may be arrested, for example, by letting the gaseous mixture to pass from the hot reaction vessel into a cold evacuated vessel or by removing the heat source from the reaction vessel with subsequent rapid or gradual cooling. In those cases where end products capable of association are formed as a result of the reaction, cooling of the reaction mixture may likewise lead to the phenomenon of contraction. Here the contraction curves will show a continuous rise as the reaction proceeds.

We were actually able to observe the contraction phenomenon during the oxidation of H_2S, acetaldehyde, propylene and acetylene, the contraction curves obtained during the oxidation of H_2S having a shape characteristic of an interme-

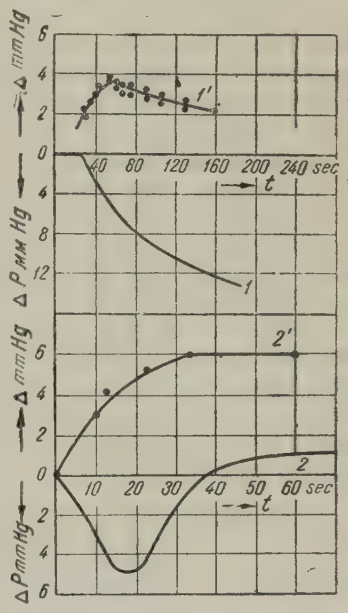
* The decrease in pressure resulting from the recombination of the atoms formed in an electric discharge at low pressures is employed, as is known, to determine their concentration. We mention here the diffusion method of Wrede, and also the method of Smallwood in which a mixture of molecules and atoms of hydrogen withdrawn from a discharge tube is subjected to rapid compression, and then the decrease in pressure owing to recombination of the atoms is studied.

diate product, and those during the oxidation of acetaldehyde, of the end product of the reaction.

The magnitude of the contraction Δ was determined as the difference in the pressures obtained in a cold vessel upon releasing (or cooling) the reaction mixture and upon releasing (or cooling) the dry air, at the same pressure and temperature, multiplied by the release coefficient for air. The values of Δ change regularly as the reaction progresses, which makes the hypothesis regarding the connexion of the contraction with the mechanism of the transformation quite plausible.

Especially striking results were to be expected in the case of the oxidation of H_2S, where according to previous data appreciable quantities of the radical SO almost unfailingly give the dimere S_2O_2 when the reaction mixture is let out into a cold vessel. This experiment is all the more interesting for us in view of the fact that with its help one may solve quite simply the long discussed problem regarding the molecular composition of sulphur monoxide and a new method of evaluating the quantity of SO in the course of the oxidation of H_2S.

The results of preliminary experiments in the study of contraction in oxygen mixtures of H_2S and CH_3CHO are given in the accompanying figure. The changes in magnitude of the contraction are compared with the change in pressure resulting from the reaction of the corresponding gases with oxygen. As is shown in a paper that is being prepared for press by V. G. Markovich and the author, in the case of the oxidation of H_2S the contraction Δ, both as regards the nature of its change and quantitatively, is in agreement with previously obtained data for sulphur monoxide SO, which is an active intermediate product of the given reaction.



1, *2*, curves of the change in pressure Δ*p* mm Hg resulting from reaction for the mixtures: *1*, $H_2S+1.5O_2$; $t=270°$ C; $p_0=100$ mm Hg; $d=40$ mm (glass); *2*, $CH_3CHO+0.75\ O_2$; $t=220°$ C; $p_0=50$ mm Hg; $d=30$ mm (quartz); *1′*, *2′*, curves of the contraction Δ mm Hg for the same cases, respectively

The curve for the change in Δ coincides well with that for the partial vapour pressure of sulphur monoxide, computed for the vapour SO—$p_{(SO)_2}$. This gives unambiguous proof of association of SO in S_2O_2 during «hardening» of the reaction mixture and illustrates the applicability of the method of contraction to analysis of the kinetics of intermediate products.

The contraction during the oxidation of CH_3CHO is evidently connected with the known fact of the association of the molecules of acetic acid forming as an end product of the reaction. In the oxidation of C_3H_6 and C_2H_2, the maximum value of Δ amounts to about 5 per cent of the initial pressure of the mixture (experiments were carried out with the mixtures: $C_3H_6-3O_2$; $t=540°$ C; $p_0=100$ mm Hg, and $C_2H_2+O_2$; $t=335°$ C; $p_0=195$ mm Hg).

It is well to note here that mere letting out of heated up H_2S, CH_3CHO, C_3H_6 and C_2H_2 does not give contraction.

Further conclusions on the nature of the reactions conditioning contraction may be made only on the basis of a detailed study of the chemism of the reaction. It is essential to analyse the composition of the mixture after release (or cooling) and seek quantitative relations between the magnitude of the contraction and the quantity of the products found. Such an analysis may give direct information as to the nature of the intermediate and end products and their concentration and, consequently, greatly facilitate the problem of building up the mechanism of the reaction (relevant experiments are now being carried out in our laboratory by Markovich, Pavlovskaya, Maizus).

In the release method it is essential to distinguish three cases: 1) slow asso-

ciation; where the time of association $t_a \gg \vartheta$, ϑ being the time of release; 2) rapid association, where $t_a \ll \vartheta$, and 3) the intermediate case $t_a \approx \vartheta$.

In the first case contraction is directly proportional to the concentration of the associating substance. In the second, the magnitude of the contraction effect is found to depend upon the volume of the cold vessel. Finally, in the third case it is necessary to take into account the kinetics of association.

Removal of the oven and a comparative study of the cooling curves of the reaction mixture and air may also lead to interesting results. If, after the oven has been removed, the main process, the effect of which cannot be taken into account, continues in the mixture left for some time in conditions of falling temperature, the method of cooling curves is not applicable. It is possible, however, after removing the oven, to effect rapid cooling (for example, by immersing the reaction vessel in ice water), and then the observed effect should coincide with the effect of the release method.

Sometimes, evidently, it is also possible to analyse slow cooling. Then from an analysis of the cooling curves it is possible to determine the temperature of the start of association, the degree and velocity of association at a given temperature, and also the energy of activation of the process.

Thus contraction is found to be a widespread phenomenon in complex chemical reactions. In individual cases a study of the phenomenon may shed light on the kinetics of the intermediate products of the reaction, their nature, and contraction, or serve the purposes of rapid determination of the concentrations of the end products of the reaction. That is why a detailed study of this interesting phenomenon is desirable.

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GEOCHEMISTRY

NATURAL GASES OF BOTTOM DEPOSITS OF MINERAL LAKES

By A. I. DZENS-LITOVSKY

(Communicated by D. S. Beliankin, Member of the Academy, 13. XII. 1944)

Any salt deposit of rock salt, sodium sulphate, potassium salts and natural soda contain a certain amount of various natural gases, the evolution and ejection of which are observed in the course of prospecting and underground mining of salts.

Investigations show that the total absolute quantity of natural gas, inclosed in the fossil salts in the form of minute inclusions, may be extremely large.

As shown by the investigations of J. V. Morachevsky and A. A. Cherepennikov, the different salts of the salt series of the carnallite of Solikamsk when recalculated to cubic metre of salt, contain 10—15 litres of gas reduced to atmospheric pressure (6). The principal gases of the salt deposits of fossil salts are hydrogen and hydrocarbons.

When prospecting the bottom deposits of mineral lakes many investigators have recorded instances of intense gas evolution. The gases evolved are confined chiefly to clastic bottom sediments, silt deposits and bottom chemical sediments, as well as to deposits of offshore bars and spits. Instances have been described where liquid mud and bottom brine were thrown out by the gas to a considerable height from drill-wells (7).

As shown by the author's field examinations of the mineral lakes of the Azov-Black Sea coast, steppe region of the Crimea, Western Siberia and Central Asia, the bottom silts and chemical sediments in most of the mineral lakes are as it were impregnated by gases throughout their thickness; in the course of drilling, and not infrequently when sinking test-pits, the brine is observed to bubble with liberated gases. In the course of this work we were largely interested in the gases of mineral lakes, whose origin is connected with the life of the mineral lake.

The principal sources of gases in mineral lakes are biochemical processes accompanying the decomposition of organic remains, and the air contained in the brine of the lake. When sinking test-pits and drilling in mineral lakes the author happened more than once to observe the liberation of gases rich in methane, hydrogen and heavy hydrocarbons, formed in the process of decomposition of organic remains upon the exposure of the silts forming the roof of the buried layers of algae.

The major part of the gases of mineral lakes represent hydrocarbons, the most often methane mixed with nitrogen. Not infrequently the hydrocarbons are under a high pressure, and their evolution proceeds in an impetuous and violent manner.

It is a common case that in mineral lakes along with ejections of gas an evolution of hydrogen sulphide is observed, the presence of which is attested beyond any doubt by its smell.

As to the composition of the gases of the bottom deposits of all mineral lakes, they are rich in light combustible constituents with an admixture of hydrogen

sulphide. When the author carried on drilling on the Saki and Sasyk-Sivash offshore bar at the Crimea, almost pure methane was evolved from drill-hole No. 7. When kindled, the gas burned with a steady flame for several hours.

The gases of chemical sediments and bottom silt deposits may differ as to their origin. Thus, the gases of the bottom chemical sediments may be entrapped by the salt during its crystallization. These gases are relatively rich in hydrogen and methane, contain heavy hydrocarbons, a small amount of carbon dioxide and over 50 per cent nitrogen.

In the specimens of bottom salts from various lakes examined by us under the microscope, among crystals of halite, hydrohalite, mirabilite and natural soda, small bubbles of gases scattered at random were clearly distinguishable. When the bottom salt was dissolved in water, the gases included in it were set free.

Owing to the diversity of the microbiological and biochemical processes taking place in the brine and bottom sediments of various lakes, the gases which are found to evolve there differ both in composition and quantity.

Part of the hydrogen sulphide, carbon dioxide and methane in the bottom deposits is produced in the process of decay and decomposition of animal organisms and algae. Considerable amounts of hydrogen sulphide are formed when the sulphates are reduced by bacteria, especially by bacteria of the genus *Microspire* whereby sulphides are converted by carbonic acid to carbonates, H_2O being displaced.

Methane or marsh gas is formed in rather considerable quantities in summer time at the bottom of most mineral lakes. The formation of methane proceeds chiefly at the expense of the decomposition of the cellulose of various algae.

The presence of nitrogen in the bottom chemical sediments of mineral lakes may possibly be explained by the fact that the salt entraps some air during crystallization. The oxygen of the air entrapped is lost on all kinds of oxidation reactions.

It goes without saying that the nitrogen in the bottom sediments of mineral lakes may be formed also at the expense of the decomposition of nitrogen-containing organic remains.

An intensive formation and accumulation of gases is often observed directly under the layer of freshly deposited salt, salt of old deposition or of underlying salt. Such an accumulation of gases was observed in Bolshoe Tavolzhan lake by P. I. Preobrazhensky.

The chemical composition of the gas collected by P. I. Preobrazhensky in Bolshoe Tavolzhan lake on September 2, 1932, was as follows (in per cent): O_2, 79.4; rare gases, 20.6; heavy rare gases, 0.374; light rare gases, 0.001. Another sample in lake Tavolzhan was found to contain 85.7 per cent O_2.

Natural gas consisting mainly of oxygen is of very rare occurrence, and in the USSR not a single instance of occurrence of such a gas has as yet been reported ([3]).

A. A. Cherepennikov explains the origin of the gas observed in Bolshoe Tavolzhan lake, which is of so unusual composition with regard to its oxygen content, by the possibility of oxygen being formed during processes of photosynthesis in the algae. Such an explanation seems quite plausible, since in the brine of mineral lakes photosynthesizing organisms are present, as shown by the biochemical studies of B. L. Isachenko.

Aside from the life of the lake, to which the gases of mineral lakes owe their origin, the gases evolved by bottom deposits of many lakes are related to the occurrence of gas in the country rocks of the lake depression. The evolution of gases in the lakes of gas-bearing regions is usually associated with gases contained in the rocks underlying the bottom of the lake depression. In these lakes the gases usually evolve from «salt holes» and «windows» in the form of gas jets or single gas bubbles escaping from these «windows».

V. I. Reineke collected in 1930 gases from salt windows—«uzyuks» of lakes Taigonur and B. Kalkaman; these gases were analysed at the gas laboratory of «Sojuzgasorazvedka» ([5]) and found to possess the following composition (in per

cent): CO_2, 0.1; O_2, 2.4; CH_4, 2.0; rare gases, 95.5; heavy rare gases, 1.402; light rare gases, an increased content (3). Lake Taigonur belongs to the type of mineral lakes, the gas regime of which is related to the gases of the country rocks underlying the bottom lake deposits (5).

Under the influence of pressure the gases migrating from the country rock underlying the lake depression penetrate along very small cracks and pores into the salt and fill up the voids and caverns of the salt formation. In a similar manner gases of country wall-rocks, surrounding from the sides the slopes of the lake depression, can also migrate to the bottom salt formation of mineral lakes.

The gases migrating from great depths of the earth penetrate to the bottom deposits of mineral lakes; part of them accumulate under the salt lens, while another fill it and find their way through it. For the mineral lakes located in gas-bearing regions this mode of gas accumulation is very frequent. A typical example of such lakes is offered by lake Soltan-Sandjar situated in the region of Pitnyak (Khorezm) and, the mineral lakes of desert landscape (2,3). The mineral lakes of desert landscape are particularly high in gas. The gases of such lakes in the Kerch Peninsula usually contain 93—98 per cent methane and 1 to 2 per cent carbon dioxide, thus presenting nearly pure-methane gases. The gases of lakes of desert landscape are associated with the country rocks underlying and surrounding the lake depressions (1).

A similar evolution of gases in mineral lakes whose origin in the main is not associated with bottom sediments of the lake, is observed in lakes Elton, Baskunchak, Inder, *etc.*

It is not infrequent that the gases issued by bottom deposits of mineral lakes have a mixed origin.

When carrying out drilling on the offshore bars of the Molochny estuary, P. K. Zamori noticed in some wells a vigorous evolution of gases which were throwing out to the surface water and mud with a strong smell of hydrogen sulphide (4).

The data of analyses show that the bulk of the gases of the Molochny estuary offshore bar consists largely of the methane, a much lower percentage falling to the share of nitrogen, and a quite insignificant, to the share of carbon dioxide and hydrogen. The content of oxygen, heavy hydrocarbons and helium is negligible.

The formation of the above gases, except for heavy hydrocarbons and helium, was due to the course of microbiological processes in marine and estuary marine silts, rich in organic material, under anaerobic conditions. As to heavy hydrocarbons and helium, these gases seem to have penetrated to the deposits of the offshore bar from deeper horizons and possibly also from the oil-bearing rocks of the region investigated (4).

Some indications as to the presence of methane in the estuary-morainic deposits of the Black Sea coast are found in a paper by V. V. Stepanov (Khadjibey estuary, Kuyalnitzky estuary and other places); unfortun tely, in this paper no data are cited on the content of heavy hydrocarbons and helium in the gases occurring in the deposits of the offshore bar of the Odessa estuary (7).

Thus it seems clear that an explanation of the origin and formation of natural gases of deposits of fossil salts should be sought for in the recent bottom deposits of mineral lakes (1).

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CHEMICAL TECHNOLOGY

ON KINETICS OF FORMATION OF MULLITE IN CLAY MATERIALS UPON BURNING

By A. J. AVGUSTINIK and E. F. USHAKOVA

(Communicated by D. S. Beliankin, Member of the Academy, 24. IX. 1944)

The materials investigated are listed in Table 1.

Table 1

		SiO_2	$Al_2O_3 + TiO_2$	Fe_2O_3	CaO	MgO	Alkali	Calcining losses
Flint clay from Dsekha	Resistant to water . . .	56.28	32.67	1.48	0.96	0.22	Not determined	9.6
Clay from Chasov-Yar	96.3 p.c. content of clay (substance with particle size below 0.005 mm) .	51.18	34.63	1.59	1.02	0.41	»	11.22
Prosianov kaolin	89.2 p.c. content of clay substance .	47.55	36.77	0.96	1.88	0.13	»	13.34

Prior to the experiments the flint clay from Dsekha was ground in a mortar and passed through a sieve of 2500 openings per cm^2. The other two materials were likewise sifted. Under equal conditions, with about 15 per cent of moisture, the samples were pressed into cylinders 7 mm high and 36 mm in the diameter to be burned in a kryptol electric furnace at 1000, 1150, 1180, 1250, 1280 and 1400° C (the temperature was raised at a rate of 4° C per min.). At the respective temperatures the samples were kept for 2, 5 and 8 hours. The process of mullitization was fixed in the samples by air hardening. Taken quickly out of the furnace the samples were at once allowed to cool in the air.

The temperature was measured with an optical pyrometer of the Holborn-Kurlbaum type, and controlled by pyroscopes.

After cooling the samples were pounded in an agate mortar. The fraction between 900 and 2500 op/cm^2 was treated in two parallel samples with a 20 per cent solution of hydrofluoric acid at 18° C for 4 hours. The undissolved residue served as indicator of the degree of mullitization.

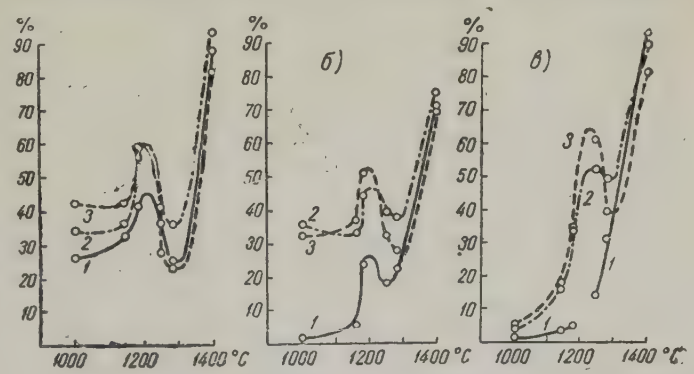
In Table 2 are given the results expressed in per cent of undissolved residue with reference to the theoretically possible content of mullite in the tested material. The latter, as derived from analysis data on the basis of the formula $3\,Al_2O_3 \cdot 2\,SiO_2$, is as follows:

Flint clay of Dsekha . . . 45.48%
Clay of Chasov-Yar 48.21%
Prosianov kaolin 51.18%

Table 2

	Burning t°C	Per cent of undissolved residue for different heat treatment intervals, *viz.*		
		2 hours	5 hours	8 hours
Flint clay of Dsekha	1000	26.2	35.1	43.3
	1150	32.1	35.8	43.2
	1180	41.5	58.7	57.1
	1250	37.1	40.9	27.3
	1280	24.9	35.8	24.6
	1400	87.9	92.5	81.7
Clay of Chasov-Yar	1000	2.15	37.3	33.4
	1150	5.18	33.6	36.3
	1180	24.8	45.6	50.4
	1250	19.0	39.8	32.7
	1280	22.2	38.1	27.5
	1400	71.8	75.2	70.1
Prosianov kaolin	1000	2.3	4.6	5.1
	1150	3.1	16.1	17.9
	1180	3.9	34.7	35.7
	1250	14.5	51.9	61.5
	1280	30.8	49.5	39.6
	1400	93.1	89.5	82.4

A graphic representation of these results is given in the adjoined figure. The curves are seen to have a maximum, which is a very important circumstance, indeed. All of the three materials tested showed this maximum at about 1200° C irrespective of the time interval in the furnace—whether 2, 5, or 8 hours. The sole exception is with the curve characterizing the process of mullite formation in the Prosianov kaolin during a 2-hour treatment. This is probably because the development of mullite in this matlaeri is retarded owing to its comparative coarseness. Its average grain-size is 0.0043 mm and so, according to Stark, it must be classed with the coarsest kaolins.



а, flint clay from Dsekha; *б*, clay from Chasov-Yar; *в*, Prosianov kaolin. *1*, 2-hour keeping; *2*, 5-hour keeping; *3*, 8-hour keeping

The discovered maximum of mullite formation may be interpreted to mean that 1200° C is the optimal temperature for the development of centres in the crystallization of mullite. As the temperature is further increased dissolution of the formed mullite crystals in the eutectic melt appears to predominate. But this stage is followed by a second rise of the curve due to the growth of the «survived» crystals. From the standpoint of chemical kinetics the process of mullite formation in the interval 1000—1300° C can be referred to the group of consecutive reactions.

For the industry the described effect is important and to be taken into account in building up a rational curve of burning for different types of ceramic ware. Keeping at 1200 C is essential for the most extensive development of crystallization centres, then, rapidly through the minimum portion, we pass into the region of mullite crystal growth at 1350—1400° C.

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GEOLOGY

A NEW REGION OF TUNGSTEN AND MOLYBDENUM MINERALIZATION IN NORTH-EASTERN KAZAKHSTAN

By P. N. KROPOTKIN

(Communicated by A. E. Fersman, Member of the Academy, 15. IX. 1944)

In north-eastern Kazakhstan, from Bayan-Aul to the Chagan River stretch Palaeozoic folds of a south-eastern strike, broken by granite intrusions. Several deposits of rare metals were discovered within the granites in 1939—1943. These deposits mark a peculiar metallogenic province adjoining from the north-east the auriferous belt of the Chingiz, Murdjik and Arkalyk ranges.

In 1939 greisens containing a small amount of wolframite and cassiterite were discovered by L. M. Afanassiev (Kazakhstan expedition of the Academy of Sciences), in the Aksan hills on the south-eastern border of the Bayan-Aul granite massif. Being explored by the Kazakhstan expedition of the «Gold Prospecting Trust» (I. A. Pauker and N. V. Yarmolovich) the area was found to contain a series of ore bodies represented by quartz-muscovite and quarzt-topaz greisen with an abundance of disseminated molybdenite and rare-earth minerals.

Seventy five km south-east of the Ak-San deposit, along the northern, south-eastern and north-western borders of the granite massif of Mt. Kalmak-Kyrgan, small veins of quartz with fluorite, scheelite and wolframite were discovered by the author in 1943.

Fifty km southward in the north-western part of the granite massif of Mt. Edrey, the author found over one hundred quartz and greisen veins, stretching in a latitudinal direction over a space of 0.5—1.5 km from the contact of the granites with Lower Carboniferous strata. Veins have also been encountered among skarns outside the massif. In thirty veins of the Edrey deposit disseminated wolframite (hübnerite) has been encountered, at places in considerable amounts. These veins with wolframite are located 2—4 km east of the quartz veins of Mt. Namaz-kyzyl-djal, in which according to M. V. Taschinina (1939) and N. D. Tikhomirov (1942) an insignificant content of scheelite has been recorded. In the gauges of the quartz veins of the Edrey deposit granites have been transformed to a quartz-muscovite greisen with wolframite, pyrite, fluorite. The veins themselves contain pyrite and secondary jarosite in large amounts, sometimes also fluorite, molybdenite, copper green.

Seven km east of the Edrey granite massif M. V. Taschinina discovered a considerable quantity of scheelite in the black sands on the Kyzyl-djar spring; and 55 km north-east of Mt. Edrey, a deposit of molybdenite near lake Kuzgan. In 1941—1942, as a result of the exploration work carried out by the Kazakh Geological Service at the Kuzgan dep sit, veins carrying finely disseminated molybdenite and a small amount of scheelite, copper sulphides, *etc.* were found among leucocratic biotitic granites near the contact of the massif. The granites in the gauges of the veins are not infrequently greisenized.

N. A. Sevryugin (of the Kazakh Geological Service) discovered in 1943 a series of quartz veins with molybdenite in granites of Mt. Koitas, 35 km south-east of Kuzgan, and small veins with an abundance of disseminated wolframite at Mt. Maili-Kara, 50 km south of Kuzgan.

Still further south is located the Deghelen granite massif, from which findings of rare metals have been reported since 1940—1941. At the northern border of the massif, amidst garnet-vesuvian and garnet-fluorite skarns there is a magnetite deposit—the Kara-djal, which had been worked in the last century and afterwards abandoned. According to some isolated analyses of samples taken by V. B. Kochurov (Kazakhstan expedition of the Academy of Sciences), the skarns carry a signi icant amount of molybdenum. As molybdenum is subject to considerable leaching in the surface samples, its content may considerably increase with depth. Mazayev has discovered in the same ores of Kara-djal the presence of tungsten.

In the north-western part of the Deghelen massif, 2 km from the contact, M. V. Studenetzky («Zolotorazvedka») discovered a series of veins of quartz-hematite greisen with quartz stringers. Both greisen and quartz stringers contain wolframite, pyrite, *etc.*; chemical analysis has shown the presence of molybdenum. Some of the thin veins are so high in wolframite that they have been worked by digger companies for several years.

Within a zone 200 km long and 50—60 km wide nearly every granite massif contains tungsten and molybdenite deposits. This zone is likely to extend 300 km to the south-east through the Chagan granite massif (where have been described manifestations of pneumatolysis), the Orda-tau and Arkat Mts., to the granites of the Aldjan and Kandagatai Mts. (180 km south of Semipalatinsk), in which showings of tungsten mineralization are known, and to the molybdenite-quartz veins of the Krzhy deposit, which has recently been discovered by a party of «Altaizoloto» in the upper course of the Kokspekta River.

The granites of all the massifs here described are of the same type. These are pink or pink-grey normal and leucocratic coarse-grained or porphyry-like biotite granites, consisting mainly of orthoclase-perthite (up to 60 per cent of the rock), quartz and albite or albite-oligoclase. They are cut by dykes of microgranites, granite-porphyries and by thin veins of aplites. In some massifs pegmatites and pegmatoid varieties of granites are encountered. The quartz veins and greisens, to which the mineralization is confined, are located, as a rule, no more than 1.5—2 km apart from the contact of the granites with the enclosing rocks. A typical feature is the presence of fluorite in the ore veins themselves, or alongside of them (quartz-fluorite veins near the ore greisens of Deghelen) and in skarns (Edrey, Kara-djal). From the skarns of Kara-djal beryl has been reported.

In so far as the majority of granite massifs, in which the above-described ore occurrences have been discovered, break through dislocated Devonian and Lower Carboniferous deposits, the mineralization may be associated with the Hercynian orogenic cycle.

It should be noted that both in age and composition the granites of the Bayan-Aul—Aldjan zone closely approach the granites of the Northern Kalba region. The Kalba zone of the tin-tungsten mineralization has also a north-western strike and is separated from the zone described by the auriferous belt of Southern Kalba.

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GEOLOGY

THE GEOLOGY OF THE LEACHING PROCESSES IN THE LOWER PERMIAN CARBONATE-SULPHATE SERIES OF THE CHISTOPOL UPLIFTS

By A. B. RONOV

(Communicated by D. S. Beliankin, Member of the Academy, 29. IX. 1944)

The family of the Chistopol tectonic elevations, which consists of three comparatively small smooth brachy-anticlines closely adjoining one another—the Buldyr-Bersut, Zmiev and Malo-Tolkish,—is located in the near-Kama part of Tataria, east of the town of Chistopol.

A structural drilling carried out here during the recent years has yielded abundant data which have not yet been fully utilized by the Geological Department of the Tatarian Geological Trust. The present paper is based on the results of a careful study of these data and is intended to shed light upon definite sides of the geological history and tectonics of the region.

§ 1. Present interrelations of rocks. The carbonate-sulphate series of the *Stafella* horizon of the Lower Permian in the Chistopol region overlies without any apparent break the carbonate deposits of the Upper Carboniferous. The boundary between the deposits of the Permian and the Carboniferous has been drawn on the basis of microfauna. The carbonate-sulphate series consists of two different lithological complexes: the lower, dolomitic, and the upper, «halogenic» one (M. S. Kaveyev). Taking part in the composition of the lower dolomitic complex are silicified dolomites poor in fauna, with inclusions of calcium sulphates, and highly altered leached rocks transformed into dolomitic flour. The latter are, as a rule, developed in those parts of the region where the superjacent gypsum-anhydrite («halogenic») complex is absent. The sections of the halogenic complex with gypsum and anhydrite contain also dolomitic bands.

From the lithological map of the region (see figure) it will be seen that the gypsum-anhydrite complex does not represent at present one continuous whole, but consists of three areas isolated from one another. The first of these is located on the eastern limb of the Buldyr brachy-anticline proper; the second, in the arch portion of the Bersut dome, and the third, on the northern limb of the Zmiev uplift.

These three areas are surrounding the field of development of rocks of similar age, which had been strongly affected by secondary processes. Here the gypsum-anhydrite complex has been eroded, and the subjacent dolomites transformed to dolomitic flour, sand and breccia. At the boundary between the unaltered and the leached series passes an intermediate zone of a joint occurrence of gypsum, anhydrites and thin bands of dolomitic flour. Thus there exists a gradual transition from the gypsum-anhydrite series to the field of development of its altered varieties.

On the western and southern limbs of the Buldyr uplift and on the north eastern limb of the Malo-Tolkish uplift the alteration zone (dolomitic flour- penetrates to a large depth, involving occasionally the upper horizons of the) Carboniferous.

§ 2. Geology and geomorphology of the space occupied by the Akchagyl deposits. The Akchagyl dark clays with intercalations of sands and sandstones rest transgressively upon the eroded surface of diverse horizons of the Permian. Rare natural exposures of these rocks are found in the Chistopol, Zmiev, Berezovka and Starochatyrcha ravines.

In the course of structural drilling the Akchagyl series was penetrated almost by all the holes on the western and southern limbs of the Bersut-Buldyr uplift, in the southern and north-eastern part of the Zmiev uplift and north-eastern slope of the Malo-Tolkish uplift (see figure).

An analysis of the distributions of the thicknesses of the Akchagyl deposits points to their zonal arrangement. The deposits increase in thickness from the north-west, where they are under 50 m thick, to the south-east; here we enter the area of very important thicknesses of the order of 200 metres and more.

The zones of equal thicknesses are stretched in a latitudinal direction, forming in the area of the synclinal downwarping between the Buldyr and Zmiev uplifts long meridional projections oriented northward.

Subsequent processes of destruction, Quartenary erosion in particular, resulted in a decrease in the original thickness of the Akchagyl deposits. These processes are especially manifest in the north-eastern extremity of the region, where the Akchagyl deposits had been washed away to a considerable extent. Although it is impossible to fill up this gap completely, some details can still be reproduced with the aid of a geomorphological analysis of the data now available. With this object in view, a map of the relief of the bottom of the Akchagyl depression was compiled. As the starting points the absolute marks of the bottom of the Akchagyl formation according to drill-holes were used. The spatial arrangement of the bottom of the Akchagyl deposits is nothing else than a cast of the bottom to which the first portions of the Akchagyl deposits were settling. In the post-Akchagyl time, there were no such intense tectonic processes as could have changed the mutual configuration of the altitude marks of the bottom of the Akchagyl deposits.

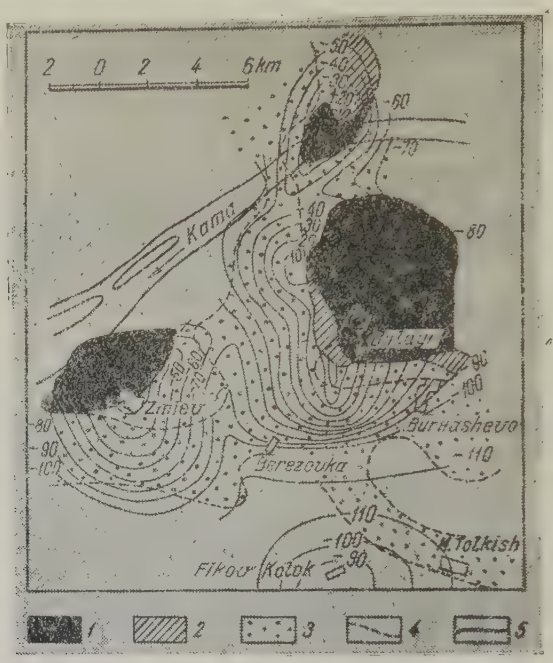
The Quaternary convulsions of the Earth's crust, as reflected in the formation of river terraces, could have been manifest only in a general monotonous elevation or subsidence of the total area.

A map of the relief of the bottom makes it possible to retrace the existence of two valleys, roughly speaking, a meridional and a latitudinal one. The former, the deeper one, passes between the Malo-Tolkish and the Buldyr elevation in a north-western direction, turns to the north along the synclinal downwarping between the Buldyr and the Zmiev elevations. The axial line of the valley is somewhat displaced east with regard to the axial line of the synclinal downwarping, and follows the western limb of the Buldyr uplift. The largest depths of the valley have been exposed in the south. The absolute marks decline more or less gradually from north to south and then south-eastward, making up on the average a drop by 4 m per 1 km. On either side of the central axial line of the valley a rapid increase in the altitude marks is observed, as a result of which in a transverse section the valley becomes a gutter-like shape, with a steeper eastern slope and a gentler western one. The maximum range of the absolute marks attains 220 m. This figure gives some idea of the depth of erosion of the underlying Permian rocks by the Akchagyl transgression.

At many points a removal of the Akchagyl deposits is noted, which points to an intense Quaternary erosion, the magnitude of which in the northern part of the region attains 60—70 m. A closely approaching figure is obtained directly from section 3 of the rotary drill-hole (Buldyr), where the thickness of the Quaternary rocks exceeds 50 m.

The second, more shallow valley stretches in a latitudinal direction along the southern slopes of the Zmiev and Buldyr uplifts. Most probably these are lateral branchings (affluents) of the main meridional valley.

§ 3. The Akchagyl erosion and leaching of underlying rocks. An analysis of the data here exposed permits of the following conclusions. The highest intensity of the erosion of the Permian rocks is confined to the axial line of the meridional valley. Here the Akchagyl deposits overlie the eroded surface of the Artinskian deposits. On either side



Lithological map of the carbonate-sulphate series (P_1) in the region of the Chistopol uplifts, *1*, field of development of gypsum and anhydrites; *2*, field of joint occurrence of gypsum. anhydrites and thin bands of dolomitic flour; *3*, field of development of strongly altered rocks transformed into dolomitic flour (gypsum and anhydrites absent); *4*, boundary of development of Akchagyl deposits; *5*, structure contours of Artinskian deposits (according to data of the Tatarian Geological Trust)

of the axial line, as they approach the borders of the valley, the Akchagyl deposits transgressively overlie higher horizons of the Permian.

The present outlines of the development of the Akchagyl deposits are more or less parallel to those of the gypsum-anhydrite series of the Lower Permian (see figure).

Some insignificant deviations of the outlines and the receding of the boundaries to 0.5—1 km are associated with the Quaternary erosion of the Akchagyl deposits. The field of development of dolomitic flour and breccias fits largely within the boundaries of the development of the Akchagyl deposits.

Along the line of maximum depths of the Akchagyl valley is passing the zone of the most intense penetration of secondary processes to depth. Here they reach the upper horizons of the Carboniferous, affecting these beds to a depth of 39 m.

On the other hand, as we approach the border portions of the Akchagyl valley, we enter the area of a poor development of secondary processes; this is the field of a joint occurrence of gypsum, anhydrites and thin bands of dolomitic flour.

The above data viewed as a whole make us conclude that the secondary processes in the dolomitic complex of the carbonate-sulphate formation (P_1) and in the upper horizons of the Carboniferous owe their origin to the sum of tectonic (transgression) and erosional phenomena which took place on the territory of this region during the Akchagyl time.

The numerous chemical analyses of the specimens of dolomitic flour, selected by the present author, carried out at the Geochemical Laboratory of the Radium Institute of the Academy of Sciences, have shown that the altered rocks of the dolomite complex are more than thirty times poorer in calcium sulphate that the unaltered varieties. The Akchagyl transgression, having eroded the gypsum-anhydrite water-resistant horizon, facilitated the access of slightly mineralized surface waters to the dolomite series. The further circulation of these waters in a direction parallel to the axial line of the valley led to a leaching of calcium sulphates with a subsequent transformation of the rocks to dolomitic flour and breccia.

The destruction of the gypsum-anhydrite cover produced also channel ways for the penetration of surface waters to deeper horizons. The consequent freshening of the Vereyan edge waters was the principal cause of the destruction of the Vereyan oil field within the deposit (1).

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MINERALOGY

ZIRFESITE—A NEW ZIRCONIUM MINERAL OF THE ZONE OF HYPERGENESIS

By E. E. KOSTYLEVA

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31 minerals have hitherto been known to contain zirconium: 1 of the group of oxides (baddeleyite), 14 silicates, comprising zircon and its diverse rare-earth varieties (malacon, cyrtolite, alvite, *etc.*), 11 zirconosilicates, minerals formed exclusively in alkaline magma, and 5 rare minerals of the group of zircononiobates and titanates. All zirconium minerals belong to rocks and pegmatitic segregations of granitic and alkaline magmas of the magmatic and pegmatitic-pneumatolytic stage of segregation. No zirconium minerals have so far been reported for the zone of hypergenesis. Of particular interest is therefore the discovery of a new mineral, zirfesite, found in the Khibina Tundras, within the zone of hypergenesis, and representing a product of alteration of eudyalite. This mineral was determined by the present author among the specimens she collected in 1926 at the endocontact zone, at a height of 200—250 m, at the contacts of Mannepachk, Khibina Tundras.

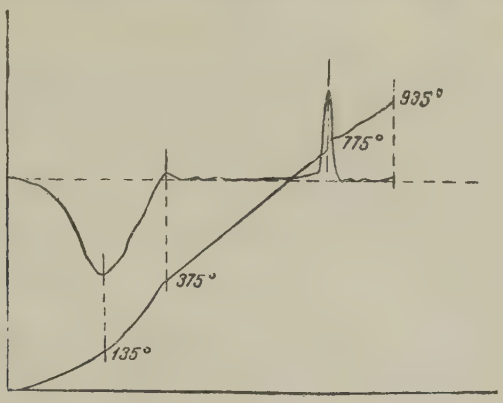
The material collected there consists mainly of hand specimens of the eudyalite-loparite vein passing along this horizon and partly sodded as a result of later soil formation processes. Zirfesite is a pale-yellow mineral, light in weight, powdery, smeary, sticking to the tongue, with a faint smell resembling that of clays.

A more thorough examination by the naked eye reveals in the powder both flakes and lamellae with a distinct pearly lustre. This flaky nature is revealed also under the microscope. The primary unaltered minerals are observed in accumulations, which is characteristic of the non-decomposed eudyalite from this deposit: small aciculate crystals of aegirine and small crystals of loparite, as well as black manganese veinlets and films of a secondary nature.

A microscopic study of the powder of the mineral in immersion liquids reveals the flaky structure of the mineral, its poor transparency, the turbidity of the flakes, and isotropism. Index of refraction $n=1.620$. The constancy of the refractive index shows a sufficient homogeneity of the composition of the mineral.

The first analyses of decomposed eudyalite, published in 1932 [1], showed the decomposition of eudyalite with a removal of Na, K, Ca, Mn, Mg, *i. e.* of all the bases, partly Si, and a relative accumulation of Zr (Hf), Fe, TR. The content of alkalies ($R_2O=6.02$ per cent) shows that the removal of these elements was incomplete, the decomposition of eudyalite being a partial one. The analysis of zirfesite (see table) shows a complete removal of alkalies and alkaline earths, a considerable removal of Si, a Zr, Fe, TR, (Nb + Ta) content much higher as compared to the original eudyalite, and a high water content. The recal-

culation of the analyses gives a result closely aprroaching the ratio (ZrO_2, Fe_2O_3) $SiO_2 \cdot n\ H_2O$, *i. e.* a type of formula closely approaching that of allophane.



Analyses of Eudyalite, Decomposed Eudyalite and Zirfesite

	Eudyalite-from Lyavochorr, analyst I. D. Borneman	Decomposed eudyalite of the contacts of Mannepachk, analyst E. E. Kostyleva	Zirfesite, analyst L. B. Tumilovich	Recalculation to molecular ratios of zirfesite
SiO_2	49.43	35.00	21.27	0.354
TiO_2	1.51	0.72	0.96	—
ZrO_2	15.30	18.87	30.47	0.249
Fe_2O_3		20.24	14.27	0.089 } 0.353
Al_2O_3	—	—	1.63	0.015
Σ Tr	0.52	—	2.12	
$(Nb,Ta)_2O_5$	—	—	2.40	
FeO	5.01	1.14	none	
MnO	0.28	3.92	0.24	
CaO	12.29	1.21	0.14	
MgO	—	5.22	0.57	
Na_2O	13.30	0.80	traces	
K_2O	—	—	0.21	
Cl	1.66	—	none	
	—		—	
H_2O^-	1.05	12.48	16.17	
H_2O^+			9.66	
Total	100.35	99.60	100.11	

$\frac{-O_2}{Cl_2}$ 0.37
99.98

The nature of this mineral is borne out by thermal analysis. The heating curve* (see figure) at the initial temperatures has at 135° an endothermal bend and a subsequent small exothermal rise, which very closely resembles the course of the heating curve of allophane and of zirconium hydroxide. With a further rise of temperature a sharp exothermal break is observed at 700°.

* Plotted by G. V. Shmakova at the Institute of Geological Sciences of the Academy of Sciences of the USSR.

Experiments on dehydration have shown that the water contained in the mineral is of adsorptive and zeolitic nature, 16.17 per cent of it being removed at 105° C, 20.96 per cent at 135°, and 23.56 per cent at 200° C. At 300° no water is left.

Thus the endothermal drop at 135° is due to the removal of the largest portion of the water. A small exothermal rise following this drop is caused by the transition of ZrO_2 from the amorphous to the crystalline variety. The sharp exothermal break at 700° is caused by the formation of a new phase. At this temperature melting of the substance takes place; the product is brick-red in colour, insoluble in HCl, and optically characterized by a high index of refraction $n = 1.720$.

The chemical properties of the mineral are as follows: in diluted HCl it is readily soluble and gelatinizes if heated. When the powder of the mineral is treated in a 5 per cent Na_2CO_3 over a water bath, a significant amount of SiO_2 is extracted.

When treated in the presence of tartaric acid, an appreciable quantity of zirconium and iron passes into solution. These determinations have shown this mineral to represent a coagulated mixture of gels of the silica, zirconium and iron, possessing properties characteristic of these gels: solubility in alkalies (for silica gels) and in alkalies in the presence of certain organic compound (for gels of zirconium and iron).

Thus the changes suffered by eudyalite in the zone of hypergenesis result in the formation of a new mineral which in its nature resembles allophane and represents a coagulated mixture of Zr, Si and Fe gels. Its formula is (ZrO_2, Fe_2O_3) $SiO_2 \cdot nH_2O$).

The name «zirfesite» has been composed from the initial syllables of the three component elements of mineral—Zr,Fe and Si.

The ZrO_2 to Fe_2O_3 ratio seems to vary in the same way as it does in eudyalite itself.

As may be theoretically presumed, formation of coagulated gels of a similar type accompanies the decomposition of other zirconosilicates and ferri- and titanosilicates, which contain in their lattice a certain amount of endocryptically entrapped Zr. Thus the existence of Zr—Si, Zr—Ti—Si gels and of a pure ZrO_2 $SiO_2 \cdot nH_2O$ may be presumed, according to the composition of the decomposing minerals and the presence of coagulated gels.

The finding of the new mineral, zirfesite, is interesting from the mineralogical and geochemical standpoint. The discovery of a zirconium mineral of the nature of allophane sheds light on the behaviour of zirconium in the zone of hypergenesis. Part of the zirconium atoms which are fixed in zirconosilicates and in the lattices of titano- and ferrisilicates, are liberated upon the decomposition of these minerals in the zone of hypergenesis in the form of minerals of the zirfesite type.

In the form of powdery-flaky aggregates of zirfesite zirconium mechanically migrates in the zone of hypergenesis, being detained in soils and silts, or mechanically transported by waters, *etc.* Under favourable conditions (alkaline mediume organic reagents) a certain quantity of zirconium may pass into solution in the form of complex compounds.

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PALAEONTOLOGY

LOPHA NOPHEPTITI sp. n. FROM THE UPPER CRETACEOUS OF TURKMENIA

By O. S. VIALOV

(Communicated by V. A. Obruchev, Member of the Academy, 8. IX. 1944)

Whilst studying the Cretaceous and Tertiary section in the Khodji-Bulak gorge (south-eastern Kopet-Dag, south of the station Dushak) the author discovered in Senonian marls a large number of highly peculiar oysters. The present paper contains a description of these oysters, which are referred to a new genus, *Lopha nophertiti* sp. n.

In the Khodji-Bulak gorge is exposed the steep northern limb of a large anticlinal fold; it is composed of Upper Cretaceous and Tertiary rocks. The Cretaceous and Lower Palaeogene beds are turned upside down. The section begins with light-coloured limestones, probably of Upper Senonian age; only the very roof of this formation is open to observation; the thickness of the formation probably attains several dozens of metres.

The formation is overlain by a packet of alternating dense cloddy grey marls and less dense clayey greenish marls (beds from 0.30 to 1.50 m). These marls contain a profusion of fauna, represented, however, by a few species only. Of *Lamellibranchiata*, *Exogyra decussata* Goldf., *Lopha nophertiti* sp. n. and *Pecten* sp. have been found; of gastropods, V. F. Pchelintzev has determined *Desmiera divaricata* d'Orb. The total thickness of the group of beds is 6 m.

The beds of marls are overlain by green clays, the thickness of which attains 70 m. It is probable from these clays that originates *Liostrea lehmanni* Rom., found in the talus.

All the beds described above belong to the Upper Senonian. The Danian is represented by a series of red clays and gypsum, alternating in its lower part with green clays. Thickness, 115 m.

The Palaeogene begins with a carbonate series of the Bukhara stage, composed of white limestones, dolomites and greenish marls (180 m). Higher up lies an areno-argillaceous series (122 m). It has not yet been ascertained whether this series belongs to the Bukhara or to the Suzak stage; the former seems more probable.

Higher up are Suzak green clays (140 m), subdivided into two parts by red clays and containing *Gryphaea* (*Phygraea*) *antiqua* Schwetz. and *Amphidonta* (*Gryphaeostrea*) *eversa* Mellev.

Judging by its microfauna the overlying series of massive green clays (450—500 m) refers to the Lower Ferghana subdivision, but it has not been possible as yet to subdivide it in more detail into the Alai and the Turkestan stages. The marine strata of the Palaeogene are overlain by a red series whose age embraces the Upper Ferghana subdivision and in a certain degree also the Chagatai division (at any rate the Massaget stage).

Let us now proceed to the description of *Lopha nophertiti* sp. n.*.

Lopha nophertiti sp. n.

The oysters described present rather massive forms characterized by the pre-

Fig. 1. *Lopha nophertiti* sp. n. (holotype), a bivalved specimen. Viewed from the side of the anterior margin. Hinge line of the two valves distinctly visible. Lower valve to the right hand side; upper to the left. Arrows indicate the orientation of the other drawings. Asterisk shows the position of the visceral portion. $^4/_5 : 1$

Fig. 2. Lower valve. View of near-visceral portion. $^4/_5 : 1$

sence of a very sharply pronounced sinus on the lower valve and a corresponding crest on the upper valve.

* The drawings are made by V. F. Kurakin from photographs.

At the external surface of the lower valve, covered by rather coarse lamellae, two portions are distinguished: a flattened upper—near apical part, rather short (Fig. 2), and an elongated lower part nearly perpendicular to the former, on which a sharp sinus is developed (Figs. 3 and 4).

If we view the hinge line of the valves, coinciding with the edges of the shell,

Fig. 3. Viewed from the lower margin. Lower valve with a well pronounced sinus is located downwards from the hinge line. $^4/_5 : 1$

we shall see in the first place an almost straight upper hinge line.

On either side of a very faintly pronounced visceral portion this line is bending at right angles (Fig. 1) and for a short space is stretching directly downwards.

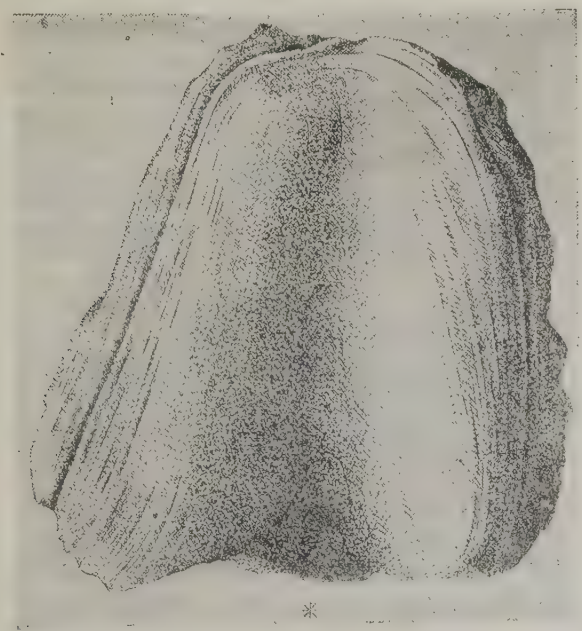

Fig. 4. Lower part of the lower valve (sinus). $^4/_5 : 1$

Further, near the lower valve it forms a smooth arc convex towards the lower edge (to the left), then a sharper arc, convex in a reverse direction, and, finally, approximately in the middle of its length, it is abruptly turning to the left and

stretches almost perpendicular to its initial direction in the form of a very smooth arc, with its convexity downwards, to the rather acute lower edge (Fig. 1).

The upper convex arc is better pronounced on the anterior than on the posterior margin. Naturally corresponding to the convex arc of the lower valve is the concave arc in the outlines of the upper valve.

At the external surface of the lower valve two swellings are distinctly marked, which are bounding a deep sinus (Fig. 2). The axial line of the sinus (*i. e.* the line of the greatest depth) is not strongly and uniformly convex; it stretches on the whole almost perpendicular to the near-visceral part of the valve.

The inner surface of the valve is almost flat; only the areas on either side of the sinus, which correspond to the sweelings of the external surface, are rather deep. The branchial notch is short, rather narrow, faintly pronounced.

The tendon notch very slightly curved, occupies less than one third of the width of the whole area. The lateral ridges are flattened. A very large impressed muscle scar is placed low and shifted towards the posterior margin of the shell. From its lower edge begins the divergence of the deepened portions on either side of the sinus.

The upper valve is flattened in the near-visceral portion but lower down a sharply marked bulge is developing upon it, corresponding to the sinus of the lower valve. In the holotype figured, this bulge has been somewhat broken off (Fig. 1).

The inner surface of the valve is flattened to the lower edge of the muscle scar. Lower follows a somewhat deeper portion—an inner sinus, corresponding to the bulge on the external surface. This inner sinus is bounded on either side by sharply prominent lobes. A very wide and low area is nearly rectangular, with a straight upper edge, over which in a hardly perceptible manner rises a small visceral portion. The median part of the area is deepened (to a greater extent than the tendon excision of the lower valve) with a prominent base: the lateral areas are flat.

I am at a loss to compare the specimens here described with any other series of oysters.

Place of occurrence: Khodji-Bulak gorge (south-eastern Kopet-Dag). Age: Upper Senonian.

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HISTOLOGY

HISTOPHYSIOLOGY OF THE MESOTHELIUM OF THE SEROSA OF SPLEEN AND TESTIS

By A. V. ANIKIN

(Communicated by A. A. Zavarzin, Member of the Academy, 29. XII. 1944)

The number of investigations devoted to mesothelium is very large; a vast majority of them are concerned with the nature of the mesothelial tissue. There is no agreement of opinion about this subject, however. Two quite opposite opinions exist as to the classification of mesothelial tissue. According to some authors, mesothelium is to be considered as a mesenchyme tissue ([1–6]), while others suggest the epithelial nature of mesothelium ([7–12]).

Investigations by Kolossov ([7]) and his predecessors have shown a complete identity of mesothelium and epithelium. Mesothelium is genuine epithelium both from the morphological and genetical point of view—this is the formula accepted by the majority of modern authors. It is also evident that the properties of mesothelium make it quite different from the ectodermal and endodermal type of epithelium, which made Chlopin to class mesothelium as a special group of coelodermal epithelium. Thus, Chlopin ([13]) advanced a new classification of epithelium, based upon its histoblastic properties. These are so stable as to be easily demonstrated in tissue cultures. According to Chlopin's data ([11]), the typical character of mesothelium is its lability and its tendency to fall out of the complex in experiments involving regeneration, and in the tissue cultures. Mesothelium displays also typical growth, both in cultures and in regeneration, when filling up mesothelium defects ([11, 12]).

There is no definite information in literature as to the function of mesothelium, except for some indirect data concerning the active function of mesothelium in the processes of transudation and resorption of serous fluid. There are also some fragmentary data as to the secreting activity of mesothelium. In his paper, devoted to the pathology of mesothelium, Borst ([14]) arrives at a poorly grounded conclusion that under definite conditions mesothelium is secreting mucus. (This author's observations were carried out on inflamed serous membranes.)

The possibility of a complicate glandular differentiation of the mesothelium in certain parts of the serous coat seemed to me quite probable.

As can hardly be doubted the body cavity of lower vertebrates being connected with the surrounding medium takes part in the excreting of products of nitrogenous exchange, like the nephrones of the kidney in higher vertebrates. This working hypothesis seemed rather promising because of the close genetic relationship between the nephritic epithelium and the mesothelium.

Thus, on account of a number of considerations the serous membrane of spleen, testis, and liver were taken as material for the investigation of the possible glandular function of the mesothelium.

By means of isolation of mesothelium with subsequent application of cytological and microchemical methods, it was possible to demonstrate the presence of complexly differentiated fields in the cells of the peritoneum, spleen and testis. They occupy the endoplasmic region around the nuclei, are spherical or hemispherical in shape, being situated at the one or at both poles of the cell.

The structure of these fields is highly variable, which points to their important functional rôle.

In the field region of each cell a Golgi apparatus can be discovered. It represents a net surrounding the nucleus, or seems to be broken into several parts arrang-

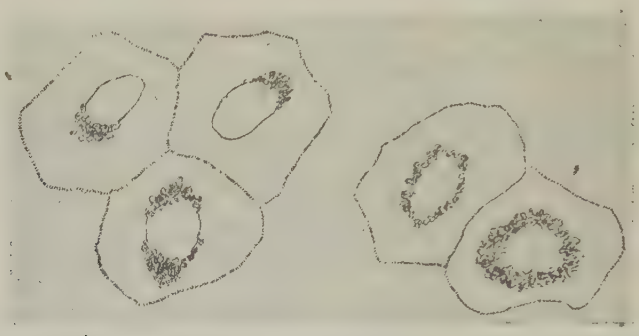

Fig. 1

ed in one or both hemispheres of the field (Fig. 1). I was successful in finding that the endoplasm regions characterized by the earliest appearance of the small granules are the very regions where the complex processes are taking place. The small granules show a reaction with sudan and, gradually increasing, are trans-

Fig. 2

formed into vacuoles with turbid contents. The vacuoles are surrounded by rings which are stained with sudan (Fig. 2).

Microchemical analysis revealed in these rings the presence of fatty acids. Further on, the large vacuoles form a foam-like structure. The rings are gradually vanishing, while the separate foam-cells fuse, to form a single vesicle. It is filled with a semitransparent fluid and protrudes above the surface of the cell. The vesicle is connected with the cell by means of a thick pedicle, which is gradually

thinning out and finally breaks off; the vesicle is then torn off into the serous cavity (Fig. 3).

After having reached a definite size, the vesicles do occasionally burst without tearing off from the cells, thus releasing their contents. After treatment

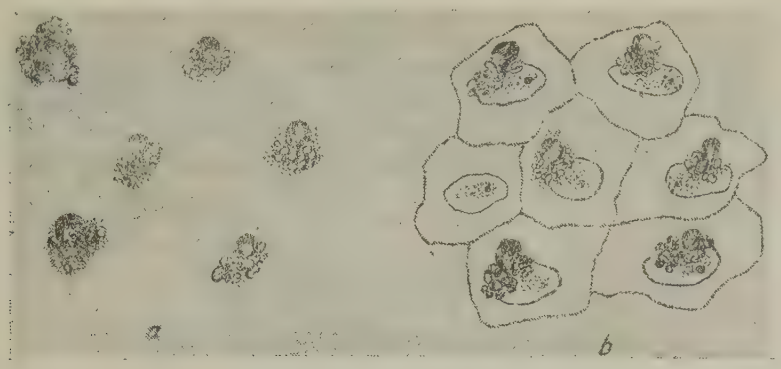

Fig. 3. Flattened section. Large excreting vacuoles formed in centres of spherical fields of mesothelial cells. *a*, immersion; high magnification and superficial focusing; secreting vacuoles seen above the cells. *b*, same magnification, focusing deeper. Nuclei and boundaries of mesothelial cells observable

of mesothelium with xanthydrol, in the region of the fields of the cells were found greenish acicular crystals, forming star-like concretions of dixanthydrol urea (Fig. 4).

The location of the crystalline star-like concretions corresponds to that of the vesicles and to that part of the spherical body in which, according to our data, is accumulated tripan-blue in the form of rosette (Fig. 5). All these morphological data, accumulation of stain, the star-like crystalline concretions of dixanthydrol-urea, whose location coincides with the rosettes of the accumulat-

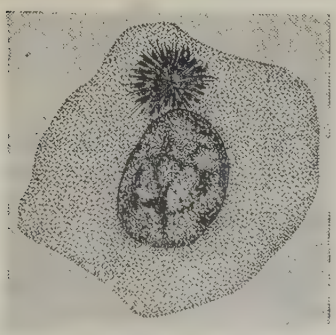

Fig. 4

Fig. 5

ed stain, the spherical arrangement of the granules, vacuoles, and Golgi's apparatus—all these point to a strict polarity of the highly specific process taking place in the mesothelial cells.

The investigation of flattened mesothelial sections permits to trace not only the rhythm of all the changes here described, but also the rhythm of this process in time throughout the mesothelial coat of an organ. The vesicles as well as the crystalline concretions are seen from the surface as figures regularly arranged on a chess-board, not all over the surface of the mesothelium, but at separate zones.

A comparison of data of several authors who studied the morphological features of the secretion process in the different types of the glandular epithelium with those concerning the structural changes of mesothelium shows that our data correspond to the most characteristic stages of secretion. The suggestion as to the glandular function of mesothelium (secretion or excretion) is also supported by

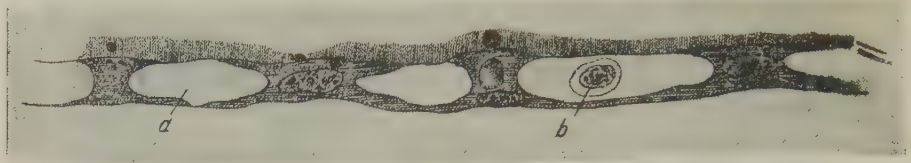

Fig. 6. Brush in mesothelial cells experimentally produced by injection of horse serum in peritoneal cavity of rabbit. Secreting vacuoles also observable. Some of them torn off into peritoneal cavity. Intense resorption that took place in this experiment led to the formation of resorption vacuoles (*a*) in the region of ectoplasm. Into one of them a leucocyte has penetrated (*b*)

the fact that in the cavity fluid of the scrotum we also found urea in a concentration exceeding that found in the blood. A series of embryological and evolutionary data as to the close relation between mesothelium and nephric epithelium, as well as the brush which was experimentally produced in the spleen mesothelium (Fig. 6) and which proved similar to that in the epithelium of the main part of kidney nephrone, makes the hypothesis of the glandular function of mesothelium highly plausible.

For lack of space we could not include in this paper all the factual material available; we report here only such data as cannot be questioned.

Conclusions. 1. The mesothelium of spleen and testis is a highly differentiated epithelium. The same seems to be true of the liver mesothelium.

2. In mesothelial cells I found complexly differentiated spherical fields located around the nuclei or at the poles of the cells.

3. The successive changes in every field are regular and characterized by features peculiar to a secretory mechanism.

4. The regular changes of the structures in the cell fields correspond to those known for the secreting and excreting mechanisms in the glandular epithelium and in particular in the kidney epithelium.

5. One of the probable glandular functions of the mesothelium of spleen and testis is the secretion of the urea.

6. The presence of a considerable amount of urea in the serous cavity of the testis was demonstrated by chemical analysis; this is in connexion with the glandular function of mesothelium.

7. We succeeded to produce a brush in the spleen mesothelium only. This structure artificially produced in the mesothelium corresponds fully to the brush of the kidney epithelium.

8. The production of a brush in the spleen mesothelium demonstrates once more that the epithelial cytoblastic potencies of the mesothelium are identical with those of the brush-like epithelium of the kidney nephrones.

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MICROBIOLOGY

INFLUENCE OF PHYTONCIDES UPON PARACHOLERAIC VIBRIOS

By T. D. YANOVITCH

(Communicated by A. A. Zavarzin, Member of the Academy, 9. XII. 1944)

The workers of the Institute for Epidemiology and Microbiology at Tomsk succeeded in demonstrating the high susceptibility of various bacteria to the effect of phytoncides, in the first place to phytoncides of garlic (*Allium sativum*) ([3]). These data went to shed additional light upon what had earlier been reported about the properties of phytoncides ([1]). Whilst investigating the effect of bactericides of vegetable origin, garlic and onions in particular, upon the causative agent of diphtheria, we succeeded in observing that the active principle was adsorbed by the microorganisms.

The present investigation was carried out with 5 strains of paracholeraic vibrios which are known to be endowed with a rather high motility. Three among them—vibrio El. Tor, vibrio Finkler-Prior and vibrio Metchnikovi—were obtained early in 1943 from the epidemiological department of this Institute, v. Metchnikovi No. 180 and v. «svetiascheysia» (luminescent), from the laboratory of Professor Zinaida Ermolieva in the same year. The cultivation properties and biochemical characters of the vibrios under investigation were found quite typical, their motility clearly pronounced. The cultures were grown on meat-peptone agar, frequently sown on peptone water. Under experiment were taken strains after 24-hour incubation and belonging to the s-form only.

Motility of vibrios as affected by garlic phytoncides. At this laboratory we have repeatedly performed experiments with vibrios in a hanging drop exposed to the influence of garlic phytoncides.

Among all the vibrios at our disposal, v. svetiascheysia (which means «luminescent») was of the minutest size and the most mobile with that. If the suspension of luminescent vibrios enclosed in a hanging drop is added a drop of garlic filtrate (after Seitz), or of garlic juice, the movement of the vibrios is not found to be stopped directly. After two minutes immobile microorganisms appear in large numbers; their number increases rapidly each minute. At the edge of the drop accumulations of vibrios are observable, while but a few are to be seen in the centre of the drop to which the phytoncides were applied. They seem to be dispersed, which should be understood as an expression of negative chemotaxis. Under high magnification clearly observable is «agglutination» of the vibrios, their arrangement into separate and rather dense groups. After 10 minutes no motile vibrios are to be found, whereas in the control drop their movement remains intense until the drop is dry. If the vibrios are introduced in the garlic juice directly, then a three minute long exposure is found to be sufficient in order that the movement of the vibrios be stopped altogether, whatever the density of the drop.

The remaining four strains of paracholeraic vibrios cease moving almost instantaneously when exposed to the effect of the volatile fractions of garlic juice.

Individual cells only continue to move about sluggishly within the first 3—7 minutes. The dependence of the moment of standstill upon the amount of the phytoncides introduced and the density of the emulsion forming the drop is found to obey a definite regularity.

The arrest of the movements of the vibrios is so typical that it may serve as a criterion for estimating the quality of the phytoncides applied. Thus the effect of garlic juice that has been preserved during a rather prolonged time (2—3 months) is apparently much below that of freshly prepared juice, nothing to say of the extract prepared by us.

Typically enough, the standstill does not lead to an immediate death of the vibrios: the fatal effect sets in after some 10—20 minutes of exposure.

Dried preparations of the hanging drop stain well with aniline dyes. The structure of the cells is not disturbed as a result of phytoncide treatment.

Susceptibility of vibrios to the volatile fractions of garlic phytoncides. The early work of B. Tokin was carried out exclusively with volatile fractions of plant extracts, which he believed to be the only carriers of bactericidal properties. In the subsequent, however, the juice of various plant has also been found to be endowed with all properties peculiar to phytoncides. It seems rather plausible that volatile fractions of garlic juice and fractions soluble in salt solutions differ as to their phytoncidal nature. Therefore, notwithstanding their value being practically the same, we undertook to test separately the influence produced upon choleraic vibrios by the volatile fraction of garlic juice and by garlic juice subjected to no preliminary treatment.

A 24-hour-old culture of choleraic vibrios was transferred in the form of a sward-like growth on to the surface of meat-peptone-agar, after which it was exposed to the effect of garlic-vapour evolved by bulbs minced to a paste. This treatment was continued for 10, 20, 30 minutes and 24 hours. After being thus treated with vapour the cultures were transferred to a thermostat. The 24-hour exposure was carried out in a thermostat. A 10-minute exposure to garlic vapour was found to bring about heavy losses among the vibrios; in the case of a 20-minute exposure, colonies were found to proliferate only at such points as were open to scarcely any vapour. A 30-minute exposure had a complete sterilizing effect upon the sward-like growth of the vibrios.

The susceptibility of paracholeraic vibrios to the volatile fractions of garlic phytoncides was additionally checked up in the following way. A small portion was removed from the agar serving as the medium for vibrios, and the spot from which it had been removed was filled up with garlic paste. The cultures were transferred to a thermostat. By the second day colonies of microbes were found to appear only at the edge of the Petri-dish opposite to the garlic paste. More than a half of the agar surface thus presented a sterile zone. Under the microscope could be identified individual vibrios scattered all over the field, whereas embryonic colonies were absent altogether. Thus paracholeraic vibrios were found to be highly susceptible to the effect of the volatile fractions of garlic phytoncides.

Susceptibility of paracholeraic vibrios to the effect of garlic juices. The adding of a 1 per cent filtrate of garlic juice to the solid nutrient medium is found to delay the growth of the vibrios by more than 24 hours, after which there appear separate colonies of vibrios which look quite dispersed as compared with the control cultures.

If a drop of garlic juice or filtrate is applied to the sward-like growth of paracholeraic vibrios, a sterile area appears at the spot. This area is found to be strictly delimited by the surrounding microbic colonies unaffected by the liquid under test.

All strains of paracholeraic vibrios available at this laboratory were tested for their response to the direct influence of garlic juice. With this aim in view 1 cm^3 of garlic juice or filtrate was added a loop of 24-hour old vibrio culture. After this, sowing was done to nutrient media at different time-intervals (directly upon the transfer of the culture, and at 10-minute intervals); in this way it was hoped to evaluate the vitality of the vibrios.

The results of these experiments are summed up in the table below.

Effect of Filtrate Prepared with Garlic Juice upon Paracholeraic Vibrios

	Strain	Sowing	10′	20′	30′	40′	50′	60′	24 hours
Results of experiment taken after 24 hours	Vibrio Metchnikovi	0	0	0	0	0	0	0	—
	» El Tor	0	0	0	0	0	0	0	—
	» luminescent	+	21 c.	0	0	0	0	0	—
	» Finkler-Prior	0	0	0	0	0	0	0	—
	Control suspension in physiological saline								
	Vibrio Metchnikovi	+	+	+	+	+	+	+	—
	» El Tor	+	+	+	+	+	+	+	—
	» luminescent	+	+	+	+	+	+	+	—
	» Finkler-Prior	+	+	+	+	+	+	+	—
Results of experiment taken after 48 hours	Vibrio Metchnikovi	+	+	+	5 c.	1 c.	0	0	0
	» El Tor	+	+	+	+	2 c.	1 c.	0	0
	» luminescent	+	21 c.	0	0	0	0	0	0
	» Finkler-Prior	+	+	+	++	+	15 c.	0	0
	Control suspension in physiological saline								
	Vibrio Metchnikovi	+	+	+	+	+	+	+	+
	» El Tor	+	+	+	+	+	+	+	+
	» luminescent	+	+	+	+	+	+	+	+
	» Finkler-Prior	+	+	+	+	+	+	+	+

Denotation: 0, sterile area; ++, intense growth, colonies uncountable; +, 30 to 50 colonies; 1—5 colonies and so on are expressed by respective figures.

The bacteriostatic and bacteriocidal influence of filtrate prepared with garlic juice upon the paracholeraic vibrios is thus made evident.

A tenfold dilution of the filtrate or juice with saline or distilled water is found to reduce the efficiency of the liquid tested; a hundredfold dilution results in a loss of bactericidal effect.

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PLANT PHYSIOLOGY

EFFECT OF SODIUM CHLORIDE UPON THE YIELD OF OATS IN THE PRESENCE OF VARIOUS SUBSTANCES (POTASSIUM BICARBONATE, ASHES, *etc.*)

By E. V. BOBKO and T. D. PRIADILSHCHIKOVA

(Communica ed by D. N. Prjanischnikow, Member of the Academy, 11. III. 1945)

Experiments by Lipman ([1]) in the USA, by Riceman and Donald ([2]) in Australia and by Aginjan ([3]) in this country have shown that the adverse influence of salts (NaCl, Na_2SO_4) upon the plants when they are present in the soil in great quantities and form solutions of great concentration, is largely due to the accompanying components present in the solution which surrounds the roots. Microelements play an important part in these processes along with macroelements. Lipman succeeded in reducing the harmful effect of the sulphates and chlorides with the aid of copper and zinc. In the experiments of Aginjan a similar effect was produced by boron or by the various combinations of balanced nutrient mixtures. One may suggest that in the case of one-sided increase in the concentration of separate salts these become harmful not because of their high concentration, but because of the low concentration of the other components of the nutrient solution, as a result of which the nutrient solution is found to be out of equilibrium.

Ivanov carried out at this laboratory in 1930 experiments on the possibility of reducing the harmful effect of sodium chloride in soil cultures by adding to the soil great quantities of ashes. These experiments gave results which went to confirm our expectation, but as the soil was an acid one, the cause of the beneficial effect of the ashes could be sought in the fact that the ash had neutralized the acidity of the soil. The experiment was therefore repeated the next year with the same crop and with the same soil (podzol-loam from the fields of the Timiriazev Agricultural Academy, Moscow). Unlike the experiment of 1939, however, the acidity of the soil was now neutralized by introducing into the pots calcium carbonate at a dose which corresponded to three quarters of the hydrolytic acidity. The basic fertilizers were introduced into the pots at the following doses: N 0.5 g per pot in the form of NH_4NO_3, P_2O_5 0.4 g per pot in the form of $NaH_2PO_4 + Na_2HPO_4$, and K_2O 0.5 g per pot in the form of K_2SO_4.

To introduce the antagonists of NaCl the following substances were applied: a) wood ashes, of needle and leaves; b) same, prealably extracted with water, and c) potassium bicarbonate. For the sake of control, additional pots were included in the experiments which contained greater quantities of P and K, to correspond to the content of these elements in the 1st and 2nd doses of ashes.

Table 2

Effect of Wood-ashes and Potassium Bicarbonate upon Development of Oats with Reference to Sodium Chloride in Soil Cultures

Sets	Yield of above-ground parts, in g per pot	Grain, in g per pot
NPK	58.3±1.7	25.7±0.7
N+PK after 1st dose of ashes	58.6±1.1	23.8±2.0
N+PK after 3rd dose of ashes	53.8±0.1	19.6±0.0
NPK+ashes, 1st dose, 120 g per pot	70.7±1.9	26.2±1.2
NPK+ashes, 2nd dose, 180 g per pot	57.6±0.3	21.1±0.8
NPK+ashes, 3rd dose, 240 g per pot	20.3±2.2	7.9±0.8
NPK+ashes, 4th dose, 300 g per pot	2.3±0.2	0.4±0.1
NPK+NaCl, 1st dose, 8 g per pot	49.8±1.3	24.8±0.6
NPK+NaCl, 2nd dose, 16 g per pot	16.2±0.6	7.1±0.2
NPK+NaCl, 3rd dose, 24 g per pot	2.2±0.3	0.3±0.0
NPK+NaCl, 1st dose+ashes 1st dose	49.4±3.2	21.7±1.4
NPK+NaCl, 1st dose+ashes 2nd dose	33.4±1.7	14.1±0.7
NPK+NaCl, 2nd dose+ashes 1st dose	27.7±2.3	12.2±0.8
NPK+NaCl, 2nd dose+ashes 2nd dose	13.8±0.5	4.8±0.1
NPK+NaCl, 3rd dose+ashes 1st dose	0.9±0.1	0.2±0.1
NPK+NaCl, 3rd dose+ashes 2nd dose	0.3±0.1	0.0±0.0
NPK+NaCl, leached ashes equivalent to the 1st dose of ashes of P_2O_5	64.5±1.5	26.2±2.2
NPK+NaCl, 1st dose+leached ashes	64.5±1.7	26.1±0.6
NPK+NaCl, 2nd dose+leached ashes	24.1±2.8	9.8±1.4
NPK+NaCl, 3rd dose+leached ashes	1.0±0.0	0.2±0.1
NPK+$KHCO_3$, equivalent to the 1st dose of ashes	66.0±0.8	24.6±0.6
NPK+$KHCO_3$, equivalent to the 2nd dose of ashes	66.0±2.7	24.0±0.5
NPK±NaCl, 1st dose+$KHCO_3$, equivalent to the 1st dose of ashes	49.9±0.9	18.1±1.3
NPK+NaCl, 2nd dose+$KHCO_3$, equivalent to the 1st dose of ashes	17.6±1.3	5.9±0.5
NPK+NaCl, 3rd dose+$KHCO_3$, equivalent to the 1st dose of ashes	1.4±0.4	0.2±0.1
NPK+NaCl, 1st dose+$KHCO_3$, equivalent to the 1st dose of ashes+ashes, 1st dose	29.9±1.1	9.8±0.6

The composition of ashes is presented in Table 1.

Table 1

Analysis of Ashes prior to and after Leaching

Kind of ashes	100 g of air-dry ashes contained		
	total alkalinity in cm^3 of 0.1 N H_2SO_4	P_2O_5	K_2O
Original non-leached wood-ash .	1076	1.67±0.10	5.22±0.12
Leached with hot water	0	2.46±0.03	0

The ashes were introduced by four doses: 1st dose, 120 g per pot; 2nd dose, 180 g per pot; 3rd dose, 240 g per pot; 4th dose, 300 g per pot. Leached wood ashes were introduced at a dose which was equivalent with respect to phosphorus to the first dose of the original ashes; potassium bicarbonate was also introduced at a dose equivalent to the first dose of the original ashes, but with respect to potassium. The pots were of glass, the amount of soil was equal to 5.97 kg of absolutely dry soil per pot. The replication was threefold. The results of this experiment are given in Table 2 (p. 517).

Below is an extract of Table 2 containing sets which confirm our suggestion as to the favourable effect of ashes as compared to the harmful action of sodium chloride (Table 3).

Table 3

Sets	Above-ground yield	Grain
NPK	58.3	25.7
NPK+ashes, 120 g	70.7	26.2
NPK+NaCl, 16 g	16.2	7.1
NPK+ashes, 120 g+NaCl, 16 g	27.7	12.2

Ashes, taken by itself, did not affect the yield of grain, but when introduced in addition to sodium chloride, they reduce the inhibition caused by the latter. Let us cite another instance relating to leached ashes (Table 4).

Table 4

Sets	Above-ground yield	Grain
NPK	58.3	25.7
NPK+leached ashes	64.5	26.2
NPK+NaCl, 8 g	49.8	24.8
NPK+NaCl, 8 g+leached ashes	64.5	26.1
NPK+NaCl, 16 g	16.2	7.1
NPK+NaCl, 16 g+leached ashes	24.1	9.8

Leached ashes, when added to basic fertilizers (NPK), increased somewhat the yield of straw having but very little influence upon the yield of grain. However, they reduced the decrease in the yield when added to NaCl taken at two doses. It was especially conspicuous with the second dose of NaCl.

Potassium bicarbonate teken at two doses produced no noticeable effect upon the yield of grain, and increased somewhat the total yield of the mass. When added to NaCl at a dose of 8 g per pot it somewhat reduced the yield of oats. Thus the balancing effect of potassium bicarbonate proved to be lower than in the case of ashes, both ordinary and leached.

It should be noted, however, that a certain fraction of the sets, for example, NPK + NaCl, 8 g per pot + ashes, 120 g per pot, behaved differently. Here the yield of the grain which was equal to 21.7 g per pot was reduced as compared to that with NaCl alone (24.8 g per pot) or with ashes alone (26.2 g per pot). It is evident that both positive and negative antagonism can be expected in the interaction between NaCl and ashes, according to the relative quantities of the interacting ingredients.

Conclusions. When high doses of sodium chloride, combined with considerable quantities of ashes, either ordinary or leached, are introduced into the soil, there are observed instances of a positive effect of paired combinations as compared with single ones. This points to the existence of a positive antagonism between the compounds which are introduced into the soil. The phenomenon in question may evidently be turned to advantage with the purpose of reducing the adverse effect of the compound which is responsible for soil salinization (sodium chloride, sodium sulphate, *etc.*).

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PLANT PHYSIOLOGY

ON THE SYNTHESIS OF RUBBER IN PLANTS

FILLING OF LATICIFEROUS VESSELS WITH FOREIGN LATEX

By A. A. PROKOFIEV

(Communicated by A. A. Richter, Member of the Academy, 22. III. 1945)

Latex varies in different plants, both in appearance and in composition. If it is rich in rubber, it is of thick consistence. Such is the latex of our native root-rubber-plants (tau-saghyz, kok-saghyz, krym-saghyz), of hevea and manihot which contain from 30 to 40 per cent of rubber. If the latex is poor in rubber, it is watery. As an example may serve the latex of the common dandelion, *Apocynum* or *Asclepias* which do not contain more than 3—5 per cent of rubber. The rapid change in the colour of the latex of low rubber concentration is evidence of a high activity of the oxidizing enzymes contained in it. Latex of high rubber content is more stable and does not change its original colour during a long period of time. This should probably be taken to mean that oxidizing enzyme systems are less active in latex with high rubber content. No less conspicuous are the differences in the content of carbohydrates, nitrogenous substances, organic acids, *etc.*

Upon coagulation the latex of the rubber-plants forms an elastic clot which consists mainly of rubber. The latex of plants poor in rubber forms a product which is lacking elasticity. Microscopic examination of latex has shown that every plant is characterized by a definite shape and size of the «globules». Non-spherical globules probably occur only in latex with a high rubber content (tau-saghyz, krym-saghyz, manihot, hevea). There exists a whole group of plants however, whose latex is rich in rubber, which possess nevertheless spherical globules (kok-saghyz, ficus, castilloa). In a latex which is poor in rubber the globules are always spherical. In young laticiferous vessels and in young individuals of rubber-plants there occur only spherical globules. It may be suggested that the accumulation of rubber in the latex of a number of plants is related to the activity of a special plastid apparatus contained in the laticiferous vessels (1).

With the aim of clearing up a number of points bearing on the problem of the synthesis of rubber and of the physiological rôle played by the laticiferous system it seemed reasonable to make an attempt to fill the laticiferous system of one species with the latex of another, characterized by a different composition of the latex. The grafting method as worked out by us (2) makes it possible to fill laticiferous vessels with foreign latex.

After having studied grafts between different *Taraxaci* we have arrived at the conclusion that latex is immobile within the laticiferous system. This conclusion was made on the ground of microscopic examination of the shape of the globules in the latex of the plants grafted. By combining plants which sharply differed in their latex we failed to find any mixing of the latexes

during the first year of the graft's life (2). However, a further study of the latex showed towards the second year that the transport of the latex from the stock into the organs of the scion was possible. Such a migration of the latex from the roots to the above-ground organs could be observed in the later part of growing season only (July and August). During the spring season (at the period of growing of the rosettes and budding) no migration of latex could be recorded. As a material for the study have served grafts between the common dandelion, kok-saghyz and krym-saghyz. In the two former plants the latex contains globules of spherical shape, while those of krym-saghyz are rod-shaped.

The method for microscopic examination of the latex has been described earlier (2). The latex was studied in the leaves and flower stalks of the scion, of the regions of the root at the point of healing (the zones of scion and stock being studied apart from each other) and of the roots newly formed by the stock. Under analysis were originally from 20 to 40 samples (for example, the latex of 40 leaves from 15—20 plants). Each sample was studied separately. As the results proved fully identical, the material could be later on reduced to 5—8 plants. The results are summarized in the table below. Grafting was done on July, 1942; the analyses were made on July-August, 1943.

Scion / stock	Shape of latex globules				
	stalks	leaves	region of roots at the points of healing		newly formed roots of stock
			region of scion	region of stock	
1. kok-saghyz / kok-saghyz	g	g	g	g	g
2. krym-saghyz / krym-saghyz	r	r	r	r	r
3. dandelion / dandelion	g	g	g	g	g
4. krym-saghyz / kok-saghyz	g+r	g+r	g+r	g	g
5. kok-saghyz / krym-saghyz	r+g	r+g	r+g	r	r
6. dandelion / krym-saghyz	r+g	r+g	r+g	r	r
7. krym-saghyz / dandelion	small rods+ +d+g	small rods+ +g	r+d	g	g

Note: g — globules, r — rods, d — dolioform. Grafting done on July, 1942, harvesting of plants for analysis, on May — July, 1943.

As may be seen from the table, the organs of the scion (leaves, stalks and the region of the root which served as a scion in grafting) contain mixed latex. In the case where krym-saghyz had served as stock rod-like globules typical of this species invariably appeared in the latex of the scion (combination 5 and 6). In those grafts where dandelion or kok-saghyz served as stocks whereas krym-saghyz served as scion, the latex was found to contain aside from globules of the krym-saghyz type, also spherical globules which

were typical of kok-saghyz and dandelion (combinations 4 and 7). Like in the first year of the vegetation of the graft in no case was there found any migration of the latex from the scion to the stock.

Such a transport of the latex from the roots to the above-ground organs may be explained by the fact that the migration of the liquids which is connected with transpiration involves also the latex and carries it along. Such an explanation seems to be rather plausible, especially on account of the fact that in the region of Burnoe (South Kazakhstan) where the experiments were conducted, violent dry winds were of constant occurrence. It is not known whether a similar transportation of the latex occurs in plants living under climatic conditions of a different character. At any rate, the fact of the mixing of the latex of the two components in the organs of the scion is evidence of the fact that there is possibility of an experimental approach to the filling, partly at least, of the laticiferous system of one plant with the latex of another.

Grafting (stock—dandelion, scion—krym-saghyz). *1*, root of dandelion; *2*, root of krym-saghyz; *а—г*, microscopic picture of latex obtained after: *а*, 2nd, *б*, 4th, *в*, 6th, *г*, tapping

The normal state of leaves and stalks which contained in their laticiferous vessels mixed latex, shows that the filling of the laticiferous system with foreign latex does not cause any visible pathological alterations. For the sake of better filling of the laticiferous system with foreign latex we also had resort to grafting. It occurs not infrequently that when dandelion is grafted onto kok-saghyz or krym-saghyz, the dandelion develops from the scion roots of its own. Such a «grafting waste» was used in our experiments intended to solve the problem. Among such grafts there was selected one plant with dandelion for scion and krym-saghyz—for stock. Grafting was done on July 23, 1942. On July 19, 1943 the plant was digged out and planted into a pot containing sand. The graft had, aside from two vigorous roots belonging to krym-saghyz, one root of dandelion which differed sharply from the former two both in colour and shape (see figure). When planting was done the end of the root was uncovered and protected from drying by a glass cap. The plant was given liberal watering and the pot was covered with a glass bell. After 48 hours following the transplantation the root of the dandelion was tapped. The latex thus liberated was subjected to microscopic examination. The incision was repeated later seven times. The tappings were spaced at 24 to 48 hours. During the whole period of the treatment and a long time afterwards the plant looked quite normal. At the time of harvest the above-ground parts of the plant dried up (the plant entered the rest period). After a week-long stay in moist sand, fresh rosette leaves started to grow.

The microscopic examination of the latex liberated by the incised root gave the following results: the latex obtained after the first tapping contained exclusively spherical globules which were typical of the dandelion. After the second tapping, minute rods typical of krym-saghyz began to appear among the globules of dandelion. The number of the rods increased along with the number of tappings (see figure). The latex which was liberated by the 7th incision (on the 11th day after the beginning of the experiment) was found to contain almost exclusively rods of kok-saghyz (see figure).

The appearance of the influx formed by the coagulated latex upon the surface of the cut of the root of dandelion has also changed conspicuously. The first portion formed a negligible film deprived of any elasticity. Such are films normally formed by coagulated latex of dandelion. The last portion produced, on the contrary, a dense, elastic film which was characteristic of the rubber of krym-saghyz. It was thus possible to «transfuse» the latex of krym-saghyz into the laticiferous system of dandelion. By tapping the roots of krym-saghyz one could fill their laticiferous system with latex of dandelion.

In order to transfer at will the latex of the one plant into the laticiferous vessels of another we have modified somewhat the technique of grafting. Roots of the young (2 or 3-month-old) plants, for example those of dandelion and krym-saghyz, were cut to pieces 4 cm long. Then, after having selected two pieces of a nearly equal diameter, we splitted each of them lengthwise, trying to make the cut exactly through the axis of the root. The halves were tightly put together and bandaged with flax fibres, with raphia or bast. Before this was done, small pieces of parchment-paper were inserted between the halves of the cutting. The paper had to prevent the growing together of the ends of the pieces and made it possible to obtain grafts where the shoots and roots of either component could easily be distinguished from each other. Rooting of the cuttings treated in the above way took place as usually ([2]).

After the development of roots, plants were obtained which had independent organs but were grown together at the middle. By tapping the roots of the one of the components one could fill its laticiferous system with the latex of the other component.

The study of the behaviour of the plants whose laticiferous system is filled with a foreign latex may throw light upon many problems connected with the synthesis of rubber within the plant, and contribute to the interpretation of the physiological rôle played by the laticiferous system.

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PHYSIOLOGY

VARIATION IN THE THERMOTAXIS OPTIMUM OF RODENTS AS DEPENDENT ON PREGNANCY

By A. I. SHCHEGLOVA

(Communicated by I. I. Schmalhausen, Member of the Academy, 1. I. 1945)

The conception of thermotaxis optimum (preferred temperature) developed within the last decade by a number of students both in this country and abroad ([1, 6, 9–11]) makes it possible to define this index as a temperature chosen by the majority of individuals within a given animal species.

As a result of these studies, the value of the thermotaxis optimum (t. o.) has been found to vary according to the physiological state of the organism. In this juncture it appeared of interest to investigate the influence of pregnancy upon the value of t. o. As seen from the data available, t. o. may somewhat rise above its normal level during the period of pregnancy ([10, 11]). This phenomenon was reported for guinea-pigs, but it failed to be borne out with sufficient clarity in house mice, however.

The present author undertook to investigate the relation between t. o. and the state of pregnancy in *Rhombomys opimus* Licht. The experiments were carried out with 6 females of identical age, whose physiological state did not deviate from the normal.

The thermotaxis optimum was determined by means of Herter's thermogradient device ([9]) which had for this purpose been somewhat modified by the author. Each female was examined 8—9 times in the course of pregnancy and twice during the period which preceded the pregnancy.

A total of 56 experiments have been carried out, which went to demonstrate that the temperature preferred by the females of *Rhombomys opimus* varies according to the period of their pregnancy. Thus, during the second half of pregnancy, and especially towards its close, a pronounced reduction in the thermotaxis optimum is observed in the majority of the females (see table).

This reduction was observed at the period specified above in all females under experiment. At the first and second period of pregnancy the thermotaxis optimum shows scarcely any alteration, like during the period that precedes pregnancy. During the first half of pregnancy one may occasionally observe even a slight rise of t. o. The data which have been reported by Herter for guinea-pigs should evidently be referred to this period.

Beginning with the third period of pregnancy (11th—15th days) a gradual, though sufficiently outspoken, downward shift of the t. o.-curve is observable *.

* The significance coefficient of the means of t. o. at the III and IV periods of pregnancy, as well as at the day of parturition, always exceeds 3.0, which points to the significance of these differences as compared to those recorded for the first half of pregnancy and for the period preceding pregnancy.

Mean Values of Thermotaxis Optimum in Pregnant Females of *Rhombomys opimus* at Various Periods of Pregnancy as Compared to those of Non-pregnant ones, in °C

No. of female	Previous to pregnancy	Periods of pregnancy				Day of parturition
		I	II	III	IV	
1	33.15	—	33.50	32.85	32.20	—
2	35.10	35.10	33.30	33.70	34.85	—
3	34.10	34.35	33.90	34.15	33.80	33.00
4	35.60	35.30	35.15	33.65	32.90	32.50
5	—	—	35.30	35.10	—	32.60
6	35.00	34.32	35.40	34.00	33.50	32.15
Mean	34.59	34.77	34.42	33.91	33.45	32.56

Especially conspicuous is this shift during the last quarter of pregnancy and at the day of parturition, in particular. The response of females to the variation in the temperature gradient varies both qualitatively and quantitatively.

In our attempts to account for the downward run of the curve of preferred temperature, plotted for *Rhombomys opimus*, we thought it correct to look for the cause underlying this phenomenon in the nature of the processes of metabolism and thermoregulation.

As well known, certain changes in thermoregulation intervene in definite animal species in the course of pregnancy, especially towards its close. For example, Murr ([7],[8]) observed in polecat females an increased sensitivity of the organism to temperature fluctuations of the external medium during the second half of pregnancy.

A close relation between the temperature of the medium and the state of pregnancy has also been reported by Eisentraut ([2],[3]) for bats. A number of investigations made with different animal species give us right to suggest a close relation existing between their t. o. and the specific traits of their thermoregulation ([6],[11]). It has been found, for example, that the settling of thermotaxis optimum proceeds under the influence of the magnitude of heat formation inherent in a given organism. An enhanced production of heat, and, consequently, intensified exchange processes, bring about a downward shift of the thermotaxis optimum.

The abovesaid makes us assume that the drop in the preferred temperature recorded at the period of pregnancy is also related to the intensification of the total exchange that takes place at this period. As shown by the numerous studies devoted to this subject, basic metabolism is found to increase in the course of pregnancy. This increase which corresponds mainly to the end of pregnancy, was reported for various representatives of mammals, and proved much similar in man and in rodents (mice, rats, rabbits). In some cases the increase in basic metabolism attained 50 per cent during the last third of pregnancy, while in other cases it was found to increase but slightly (by 10 per cent) if at all. The variation in the response of pregnant females to the temperature gradient may well be accounted for by similar individual fluctuations of the basic metabolism.

Among the arguments advanced to explain the increase in basic metabolism at the period of pregnancy the most plausible seems to be the assumption made by some writers ([4],[5]) as to the endocrine, or more strictly speaking, thyrogenic origin of this phenomenon. This assumption seems to be supported by the sharp increase in the basic exchange towards the close of pregnancy, its rapid drop to the normal level following parturition, and the individual fluctuations in the increase of metabolism. While experimenting on rabbits, Sachs, Leibson and

Likhnitzkaya ([5]) found basic metabolism at the close of pregnancy to be accompanied by a stop in the increment by weight, or even a certain decrease in the weight of the females. The authors are inclined to interpret their observations as evidence of a hyperfunction of the thyroid gland, taking place at this period.

A similar run in the variation of body weight was recorded by us for *Rhombomys opimus* Licht. in which the weight was found to become stable or slightly to decrease within several days before parturition.

The fact that the intensified endocrine activity of a pregnant organism calls forth an increase in the basic exchange and, as a result, is followed by the changes in thermoregulation described above, is also corroborated by these phenomena being present in the case of artificial hyperthyrosis ([12]).

From what has been said above we feel authorized to conclude that the downward shift in the thermotaxis optimum observed in *Rhombomys opimus* at the period preceding parturition is caused by the thyrogenic alterations that intervene in the general metabolism and thermoregulation of these animals during the period of pregnancy.

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EXPERIMENTAL MORPHOLOGY

REGENERATION OF EXTREMITIES IN *PERIPLANETA ORIENTALIS*

By L. ZHINKIN

(Communicated by A. A. Zavarzin, Member of the Academy, 9. XII. 1944)

When studying regeneration of the muscle tissue in the black-beetle *Periplaneta orientalis* I was successful in observing atypical regeneration of extremities. The leg as a whole is not capable of regeneration in *P. orientalis*, although all the tissues constituting it are able to regenerate. It has already been shown in my previous investigation of this problem (1) that in *Crustacea* the extremities were found able completely to regenerate while in the black-beetle the extremities were not regenerating completely but in an atypical way. In that investigation I had the opportunity to compare the changes taking place in regenerating tissues and to accumulate data as to the mechanism underlying the loss of regeneration ability in insects. This comparison also permits to make out the rôle of the different tissue elements in the process of the formation of the regenerate.

The observations were carried out with two black-beetles in which the legs were autotomized under natural conditions. Microscopical examination of the material showed the presence of multiple regenerates, microscopical in size. The age of these regenerates was unknown. Both regenerates were found nearly identical in composition, in one of these individuals the regenerates were developed more strongly than in the other. The structure of the more vigorous regenerate was as follows: after autotomy there remained only the coxa which served as a basis for regeneration. The free space under the chitin was occupied by three regenerates. Out of these, the more strongly developed one seemed to be the continuation of the leg. The remaining two were located one behind another, by the side of the larger one. The latter possessed all the parts normally present in a leg, *i. e.* the trochanter, femur, tibia and tarsus. All these parts were developed quite distinctly, and their identification offered no difficulties. Regenerated muscles were found to be present in the trochanter and femur. They developed from myoblasts; nerve fibres were observed among the regenerated muscles of the femur.

The free space in the regenerate was mainly occupied by rounded basophylous amoebocytes. The latter, few in number, were also found between the muscles of the first two joints of regenerate. In a larger number thy were present in the subsequent joints of the regenerate being the only cellular elements within the latter.

In addition to the round amoebocytes there were also spindle-like amoebocytes and those supplied with several processes. The latter were present in a small number and seemed to be transformed into desmoblasts.

The second regenerate also consisted of a number of joints. Of these four were well developed. The tarsus was apparently not divided into joints. The hypo-

derm consisted of a single layer of cells with large and bright nuclei and basophilous protoplasm. Between the joints the hypoderm consisted of several layers. The whole cavity inside the second regenerate was occupied by loosely packed amoebocytes. The second regenerate was poorly developed as compared to the first one. It contained neither muscles nor nerve fibres. Above and by the side of the second regenerate was located the third regenerate. It had the shape of a lengthened sack-like outgrowth and was not divided into joints. Its wall was made up of one layer of thickened hypodermal cells.

Just like the second regenerate this one was filled with amoebocytes. These observations seem to mean that the leg of an adult black-beetle is to some degree endowed with regeneration ability. In order to secure some data as to the process of formation of regenerate a series of experiments was carried out. The legs in an adult black-beetle were amputated at different levels, most frequently at the middle of the femur and the joint between the coxa and trochanter. In the former case autotomy of the leg was often observed.

The legs of the experimental black-beetle were fixed at different periods after amputation, not exceeding 60 days. The latter term was found to be insufficient, however.

In most of the experimental animals epithelization of the wound by hypodermal cells was complete after 5—6 days. The same rate of epithelization was observed by Lazarenko (2) for unicorn-beetle. By the 30 day after operation regenerates begin to form in the majority of experimental animals. At the place of the formation of the regenerate the hypoderm becomes multilayered. In some specimens small outgrowths are formed by that time. They consist, as a rule, of 2—3 joints. In some cases, however, they look like simple sack-like outgrowths. In the majority of cases there are formed 2—3 regenerates located at a certain distance from each other. Out of these regenerates one is more often located at the external and another at the internal edge of an extremity. Each of them develops quite independently. They are filled with amoebocytes among which myoblasts were but rarely found. In general the regeneration picture varied as to the individual.

If the amputation was practised between the trochanter and coxa, the latter served as a basis for regeneration. In the majority of individuals there were formed 2—3 regenerates similar to those found in nature. In this case one of the regenerates was developed more strongly than the rest. If the amputation was done through the middle of a joint, two small regenerates were formed. They were equally developed and poorly differentiated. Along with the cases just described, individuals were observed which had no traces of regeneration, except for the epithelization of the wound. In case regeneration took place, 2—3 regenerates were found to develop. They were microscopical in size. The whole process of formation of regenerates seemed to be strongly inhibited and the regenerates formed were rudimentary ones. In addition to the blood cells only single myoblasts were found inside the rudimentary regenerates.

The observations here described give some idea as to the regeneration ability of the black-beetle. A comparison of these observations with those on the regeneration of the extremities in crustaceans makes it possible to draw an interesting conclusion as to the cause responsible for the loss of regeneration ability in insects.

In the black-beetle, as well as in crustaceans all the tissues are capable of regeneration. At the same time the extremities in crustaceans are regenerating completely whereas in the black-beetle they are not able to regenerate. Difference in the regeneration ability may be due to a disturbed correlation between the tissues forming the regenerate, or to a limited regeneration of a definite tissue of the regenerate.

As has been shown by Copeč (3) in the case of leg denervation in *Asellus* there is formed a leg deprived of musculature but quite normal in appearance. This observation seems to show that the hypoderm forms the regenerate without participation of any other tissues.

Therefore, the developmental potencies in *Asellus* are entirely confined to the hypoderm.

This conclusion may probably be extended to all the *Arthropoda*, since it is in good agreement with the facts on the embryology and regeneration in *Arthropoda*. It is possible to admit, therefore, that the limited regeneration ability in the black-beetle is connected with the limited morphogenetic potency of the hypoderm rather than with the disturbance of the correlation between tissues forming the regenerate.

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ZOOLOGY

A NEW GENUS OF *CEPHIDAE (HYMENOPTERA)* FROM TADJIKISTAN

By V. GUSSAKOVSKY

(Communicated by E. N. Pavlovsky, Member of the Academy, 16. I. 1945)

The author gives a description of a new genus and species of *Cephidae*, found by Dr. A. Stackelberg in spring 1944, in a woody zone of the southern slope of the Ghissar mountains in Tadjikistan.

This insect seems to be a very interesting one in many respects. It belongs to the subf. *Hartigiinae*—a group, represented till now in the Central Asia fauna by 2 species only, and confined in its biology especially to localities rich in tree and shrub vegetation. Thus it is, most probably, an endemic form typical of the woods of the Ghissar mountains. As to its characters the genus seems to occupy an intermediate position between *Hartigia* Schdte. and *Syrista* Knw., closely resembling the latter genus in colour, position of the cubital veins and some other characters. This permits, as it seems to me, to determine the zoogeographical character of the form: *Syrista* Knw. is mainly an oriental genus with a single species known from Mediterranean Europe, most probably, also a relict of oriental origin as well as many other, truly oriental species of *Hymenoptera*, found there by me, *e. g. Sceliphron deforme* Smith, *Crabro* (*Thyreopus*) *pugillator* Kohl, *C.* (*Th.*) *tuberculifer* Kohl (*Sphecidae*), or some endemic species, belonging to the oriental groups,—*e. g. Pison hissaricum* m. of the subg. *Pisonoides* Sm., represented in the Palaearctic fauna, besides this species, by a single species from Corea, or *Trypoxylon sogdianum* m., a member of the *T. bicolor* Sm. group—one of the most peculiar groups of tropical Asia, comprising only a few species, entering Palaearctic Asia in the East China, Japan and Ussuri-Land. I consider the existence of such faunistic elements to be one of the most characteristic features of the fauna of this part of Middle Asia.

HISSAROCEPHUS gen. nov.

Belongs to subf. *Hartigiinae*, and is closely allied to *Syrista* Knw. and *Hartigia* Schdte., differing from both in some characters of venation, structure of antennae, and from the former genus also in the form of the saw-sheath and armature of legs.

Body long and slender, with abdomen twice as long as head and thorax taken together. Head not narrowed behind the eyes, as in *Hartigia* Schdte., anterior margin of clypeus symmetrical, somewhat prominent and truncate in the middle. Clypeus on each side with deep impression above the margin, while median part longitudinally convex, lateral margins prominent, nearly forming straight, elevated ridges on its sides. Genae as long as antennal scape, with a deep, semicircular foveole below, above the base of mandibles; temples, viewed from the side, somewhat broader in the upper part than the eye, finely marginated posteriorly

hind ocelli on the level of the hind eye margin. Pronotum as long as the breadth of its posterior margin, the latter deeply, angularly excised, as in *Hartigia* Schdte., mesonotum and scutellum as well as mesopleurae with a distinct episternal suture, separated below from sternum by fine sutures, beginning from the middle coxae and converging anteriorly. Abdomen long and slender, with the 1 tergit cleft and separated from the 2 one by a broad emargination, filled with a whitish membrane, the 2 tergit slightly sinuate in the middle of the apical margin; apical segments almost as in *Hartigia* Schdte., the ultimate one with a distinct median area, limited laterally by fine but distinct sutures, reaching hind margin at the very base of cerci; saw-sheath obtusely triangular, much broader than in *Syrista* Knw. and not curved downwards, twice as long as cerci, viewed from above, not perfectly closed basally. Antennae a little longer than head and thorax united, rather thin, but distinctly thickened in the middle, 29-jointed, with basal joints not flattened; the 3 joint distinctly longer than the 4, the following ones not crenulate beneath, and, from the 7 on, broader than long. Legs as in *Hartigia* Schdte.—middle and hind tibiae with only 1 praeapical spur, claws cleft, or, strictly speaking, with a very oblique denticle, much before apex and much shorter than the apical one. Alar venation as in *Hartigia* Schdte, but base of cubitus much higher—near the very point of junction of the basal vein and subcosta, the 1 cubital vein a little postfurcal, the 2 one interstitial. Coloration peculiar, black and rufeus, without typical yellow pattern of *Hartigia* Schdte.

Genotype—*Hissarocephus stackelbergi* sp. n.

Hissarocephus stackelbergi sp. n.

Female. Black, mandibles somewhat reddish-brown towards apex, membrane between the 1 and 8 tergits yellowish-white, the 3 tergit with corresponding sternit ferruginous; apices of anterior and middle femora, as well as tibiae and tarsi of all legs of the same colour, except 2 last tarsal joints, blackish; wings darkened, nearly light fuscous, with black veins and pterostigma, costa and subcosta towards base, as well as intercostal cell between them brownish-ferruginous. Head and thorax covered with dense, but very short, black hairs, clypeus shining, front opaque, finely rugulose, with a not deep, but distinct rounded foveole above antennae, and a rather broad, but quite plane depression with somewhat elevated margins before the anterior ocellus, resembling «area pentagona» of many *Nematini*; vertex rather shining with fine, but distinct, moderately dense puncturing. Thorax with the same short, black hairs and fine, dense puncturing, less distinct, somewhat rugulose on pronotum and mesopleurae; median lobes of mesonotum divided by a short, deep middle furrow, their outer margins converging at a comparatively obtuse, almost right angle with the apex protruding and sharp; scutellum nearly flat, not distinctly delimited, with somewhat stronger and less dense punctures and almost opaque, finely rugulose interspaces, 1. tergit densely rugulose, the following ones finely and densely puncturate, with only a faint lustre; cerci rather long, somewhat thickened and oblique apically, saw-sheath twice as long as cerci, rather broadly triangular, with apex roundly obtuse and upper margin slightly arched, but not curved downwards: structure of antennae, as well as characters of legs and alar venation see above, in generic diagnosis. 17 mm. ♂ unknown.

Tadjikistan, southern slope of Ghissar mountains, Kondara valley, 30 km north Stallinabad, 1100 m, 3. V. 194, 1♀(A. A. Stackelberg). Type in the collection of the Zoological Institute, Academy of Sciences of the USSR.

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